A Note on Tuning the Gradient Wind Relation Currently Used in the LFM

Edward A. O'Lenic
Development Division

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This is an unreviewed manuscript, primarily intended for informal exchange of information among NMC staff members.
Introduction

An effort to choose a wind relation to calculate winds from heights for use in the Optimum Interpolation system (O/I) in regions of sparse wind observations, has prompted the evaluation of various wind relations during the past year. The simplest wind relation to incorporate curvature effects is the gradient wind equation, a modified version of which is now used in the analysis portion of the LFM (Brown, 1971).

Background

The gradient wind relation under investigation may be derived from the invariant form of the equations of motion in component form, in pressure coordinates,

\[
\begin{align*}
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(\phi + \frac{u^2 + v^2}{2}) - (\zeta + f)v + \frac{\omega \partial u}{\partial p} &= 0 \\
\frac{\partial v}{\partial t} + \frac{\partial}{\partial y}(\phi + \frac{u^2 + v^2}{2}) + (\zeta + f)u + \frac{\omega \partial v}{\partial p} &= 0 \\
\end{align*}
\]

where

\[
\begin{align*}
\phi &= gZ \\
g &= 9.8 \text{ ms}^{-2} \\
Z &= \text{height of pressure surface} \\
u &= \text{eastward component of wind} \\
v &= \text{northward component of wind} \\
\zeta &= \text{relative vorticity of wind} \\
f &= \text{coriolis parameter} \\
\omega &= \frac{dp}{dt} \\
\end{align*}
\]

Assuming \( \omega \)-terms are negligible and letting \( \partial \phi / \partial x = f v_g \), and \( \partial \phi / \partial y = -f u_g \), where \( u_g \) and \( v_g \) are the geostrophic wind components, and \( K = \frac{u^2 + v^2}{2} \), we get

\[
\begin{align*}
\frac{\partial u}{\partial t} + f v_g + (\frac{\partial K}{\partial x} - v \zeta) - f v &= 0 \\
\frac{\partial v}{\partial t} - f u_g + (\frac{\partial K}{\partial y} + u \zeta) + f u &= 0 \\
\end{align*}
\]
Assuming that the horizontal accelerations $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}$ may be absorbed into a constant factor, $k_1$, applied to the non-linear term, and further, that vorticity and kinetic energy may be approximated by their geostrophic counterparts, we get:

$$u_\ast = u_g - \frac{k_1 k_2}{f} (\frac{\partial k_g}{\partial y} + u_g \zeta_g)$$

$$v_\ast = v_g + \frac{k_1 k_2}{f} (\frac{\partial k_g}{\partial x} - v_g \zeta_g)$$

where $u_\ast, v_\ast$ = gradient wind components,

$$k_2 = \frac{\sin L - \sin 20}{\sin 30 - \sin 20}, \quad 0 \leq k_2 \leq 1 \text{ for } 20^\circ \leq L \leq 30^\circ.$$  

$k_2$ is a latitude weighting parameter which has been inserted in equations (3) to facilitate a gradual transition from gradient winds northward of $30^\circ$N, where $k_2 = 1$, to guess winds south of $20^\circ$N, where $k_2 = 0$. The parameter $k_1$, is the subject of the efforts described in the next sections. In vector form, equations (3) become

$$w_\ast = w_g + \frac{k_1 k_2}{f} (\mathbf{k} \times \nabla k_g - w_g \zeta_g)$$

Procedure

Seven synoptic cases were chosen from which to compute gradient winds (Appendix A). For each case, 10 sets of winds, each using a different value of $k_1$, were calculated and verified against observations using the SUMAC verification code. Verification statistics were compiled for the 110 station NA110 area and the 96 station EUR96 area (Figure 1).

Results

Graphs of RMS vector wind deviation and bias (gradient versus observed) versus $k_1$ (Appendix B) show that, while the RMS generally reached a minimum for $0.10 \leq k_1 \leq 0.25$, the gradient winds exhibited a negative bias which decreases monotonically with increasing $k_1$. Further, this bias has its largest magnitude and most rapid rate of decrease at 250 mb, where it varies from about -1 ms$^{-1}$ to -3 ms$^{-1}$ for $0.05 \leq k_1 \leq 0.33$. This behavior of the bias indicates that the non-linear term, $\mathbf{k} \times \nabla k_g - w_g \zeta_g$, was negative below 100 mb for all values of $k_1$ for all 7 cases in the test. Thus, the effects of cyclonic curvature apparently dominated the

* NOTE: Hough height analyses were used.
Figure 1. SUMAC verification areas NA110 and EUR96.
results of this test. Ideally one would want to choose cases in which
cyclonic and anticyclonic curvature were distributed in equal amounts
among troughs and ridges, respectively. Equation 4 implies that, for
such cases in the northern hemisphere, the gradient wind exceeds the
geostrophic wind in ridges to the north of the jet, and lags the geo-
rophic wind in troughs to the south of the jet (Figure 2). Graphs of
gradient wind RMS error and bias were also plotted (Appendix B) for the
96 station EUR96 area (Figure 1). The upper air height gradients in
this area during May were weak in contrast to those over NA110. The RMS
vector wind deviation and the bias of the gradient winds were considerably
lower over EUR96, and the rapid decrease in bias with increasing k, observed
over NA110, is only weakly present.

In view of the tendency for the RMS vector wind deviation (gradient
vs observed) to reach a minimum for $0.10 \leq k_1 \leq 0.25$, along with the large
increase in negative bias for increasing values of $k_1$, the value $k_1 = 0.15$
at all mandatory levels from 850 to 100 mb was chosen as the optimum
value of $k_1$. As a test, winds were computed from a 12Z 21 October
1979 Optimum Interpolation (O/I) height analysis. To verify that this
change in $k_1$ improves the details of the wind fields, winds computed
using the old values, $k_1 = 0.2, 0.2, 0.3, 0.3$ at 850, 500, 250, 100
mb, respectively, are shown in Figures 3 and 4, while winds computed
using $k_1 = 0.15$ at each level are displayed in Figures 5 and 6. Observa-
tions are shown in Figure 7 and 8.

Comparing the gradient winds (Figures 3-6) with observations
(Figures 7, 8) reveals that the gradient winds reproduce the overall
details of the observed wind field quite well. Further comparison how-
ever, reveals that winds computed using $k_1 = 0.30$ (at 250 mb) were consis-
tently weaker than those observed in troughs by from 20 to 110 percent,
and stronger than observations in ridges by up to 40 percent. These
characteristics are most marked in the trough-ridge-trough pattern extend-
ing from 45N, 145W into the western United States (Figure 3), in the
broad trough over north central Europe (Figure 4). Notable differences
between gradient winds and observations in the opposite sense occur in
the ridge over the eastern U. S. (Figure 3), in the short-wave ridge
near Iceland (Figure 4), where gradient winds are slightly weaker than
observed, and in the trough over Tripoli (Figure 4), where gradient
winds are stronger than observed. Gradient winds calculated using $k_1 = 0.15$
(Figures 5, 6) improve upon those calculated using $k = 0.30$ most
notably in the trough-ridge-trough pattern west of 100W, in the trough
north of Cuba (Figure 5), and in the trough along 30W, southeast of
Greenland (Figure 6). Changing $k_1$ from 0.30 to 0.15 also reduces the
winds in the ridges, but only 5 to 10 percent in most cases, and then
only in sharp ridges. Large scale features, such as the broad trough
over north central Europe (Figures 4, 6) remain unchanged by this change
in $k_1$. 
Figure 2. Physical interpretation of equation 4.
Figure 3. 250 mb height analysis North America gradient winds, $k_1 = 0.30$
12Z 21 October 1979
Figure 5. 250 mb height analysis, North America, gradient winds, $k_1 = 0.15$
12Z 21 October, 1979
Figure 7. 250 mb wind observations, North America, 12Z 21 October, 1979.
Figure 8. 250 mb wind observations, Europe, 12Z 21 October, 1979
Summary and Conclusions

A version of the gradient wind law currently in operational use at NMC has been studied in order to assess the usefulness of modifying one of its parameters, and its possible usefulness for providing winds to the O/I. A series of tests indicated that the RMS vector wind deviation (gradient vs. observed) for winds computed from this relation reached a minimum for values of the parameter $k_1$ of $0.1 < k_1 < 0.25$. The 250 mb mean speed error, or bias, was negative for all cases in both the NA110 and EUR96 areas. Further, this bias became increasingly negative with increasing $k_1$, varying from about $-1.0$ ms$^{-1}$ for $k_1 = 0.05$, to $-3.0$ ms$^{-1}$ for $k_1 = 0.33$. Considering these RMS and bias variations, it was decided that a value of $k_1 = 0.15$, instead of the value $k_1 = 0.30$, currently used at 250 mb in the LFM, might result in improved gradient winds, especially with respect to their mean speed bias. Comparisons of winds calculated using both values of $k_1$ with each other, and with observations, confirmed that winds calculated using the updated $k_1$ brought wind speeds in most troughs into better agreement with observations, with only slight decreases in wind speeds in ridges. This certainly can be regarded as an improvement, at least for the 12Z 21 Oct 79 case. Table 1 lists the value of $k_1$ currently used in the LFM and gives a suggested list of updated values for all standard levels based on the results of this study.
Table 1. Current and Suggested Values of Parameter $k_{1}$ in LFM

<table>
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<th>Pressure level (mb)</th>
<th>Current Value</th>
<th>Suggested Value</th>
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<tr>
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<td>0.0</td>
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<tr>
<td>850</td>
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<tr>
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<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>500</td>
<td>0.2</td>
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</tr>
<tr>
<td>400</td>
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<td>0.15</td>
</tr>
<tr>
<td>300</td>
<td>0.2</td>
<td>0.15</td>
</tr>
<tr>
<td>250</td>
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<td>0.15</td>
</tr>
<tr>
<td>200</td>
<td>0.3</td>
<td>0.15</td>
</tr>
<tr>
<td>150</td>
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<td>0.15</td>
</tr>
<tr>
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<td>70</td>
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<tr>
<td>50</td>
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<td>0.1</td>
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References

Acknowledgements

I wish to express appreciation to A. J. Desmarais for providing both a coded version of the gradient wind equation, and the impetus to carry out this study. Conversations with Dr. John A. Brown clarified many questions. John R. Ward supplied the height-analysis for the 12Z 21 October 1979 case.
APPENDIX A

250 mb Height-Isotach Analyses
APPENDIX B

RMS vector wind deviation and bias from SUMAC verifications
(gradient winds vs. observed winds)
Graph: RMS Vector Error and Bias

vs Gradient Wind Modulation Parameter M, for
12/05 May 90, NA10.

<table>
<thead>
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<th>ERR (m/s)</th>
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<th>100</th>
<th>500</th>
<th>950</th>
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<td>1</td>
<td>0</td>
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Graph: RMS Vector Error and Bias

vs Gradient Wind Modulation Parameter n, for
12/05 May 90, NA10.

<table>
<thead>
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<th>500</th>
<th>850</th>
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<tbody>
<tr>
<td>ERR (m/s^2)</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
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k_1