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Computation of Mandatory Level Heights Given σ Layer PE Forecasts of Heights and Temperatures (A Revised Method)

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COMPUTATION OF MANDATORY LEVEL HEIGHTS
GIVEN σ LAYER PE FORECASTS OF HEIGHTS AND TEMPERATURES
(A REVISED METHOD)

The basic procedure, which is not altered, is to take the hydrostatic equation in the form

$$\frac{\partial gz}{\partial \pi} = - c_p \theta \tag{1}$$

$$\pi = \left(\frac{p}{P}\right)^{R/c_p} \qquad P = 1000 \text{ centipascals} \tag{2}$$

and integrate it from a σ surface of known z and π to a mandatory level of known p (and therefore π) thus obtaining the value of z on the mandatory level. In order to do this mathematically, it is necessary to make a statement relating θ and π - what has been done (and will continue to be done) is to assume a linear variation of θ with π:

$$\theta = a + b(\pi - \pi_o) \tag{3}$$

π_o is a particular known value of π at which θ is known and thus $a = \theta_o$; $\theta_o - \pi_o$ is a reference point to which to tie the linear variation; b represents the slope $\Delta\theta/\Delta\pi$ of the θ-π line and requires known values of θ and π at two levels for its specification. And therein lies the difficulty--what levels do we select from the forecast σ-coordinate information to specify the values of a and b?

But first let us substitute (3) into (1) and do the integration from a σ surface of known z_σ to the desired pressure surface to obtain the wanted z_p. The result is

$$z_p = z_\sigma - \frac{c_p}{g}\left(a(\pi_p - \pi_\sigma) + \frac{b}{2}\{(\pi_p - \pi_o)^2 - (\pi_\sigma - \pi_o)^2\}\right) \tag{4}$$

where π_p and π_σ are the π values at the known pressure and σ surfaces respectively.

Returning to the question of how to specify a and b (and θ_o, π_o), recall that the PE models (either 6- or 8-layer) forecast temperature and pressure difference in or across layers. From these, it is straightforward to compute height and pressure (and π) at σ levels. This then constitutes our σ coordinate information: z and π at σ-levels, and θ in the layers between. As it is necessary to assign a value of π to the middle of the layer and attach our layer θ to that value, the mean value of the π's at the adjacent levels is computed for this purpose. (See Tech. Proc. Bull. #11, 13 Feb. 1968)

Fig. 1 illustrates the situation (in the vicinity of the tropopause)

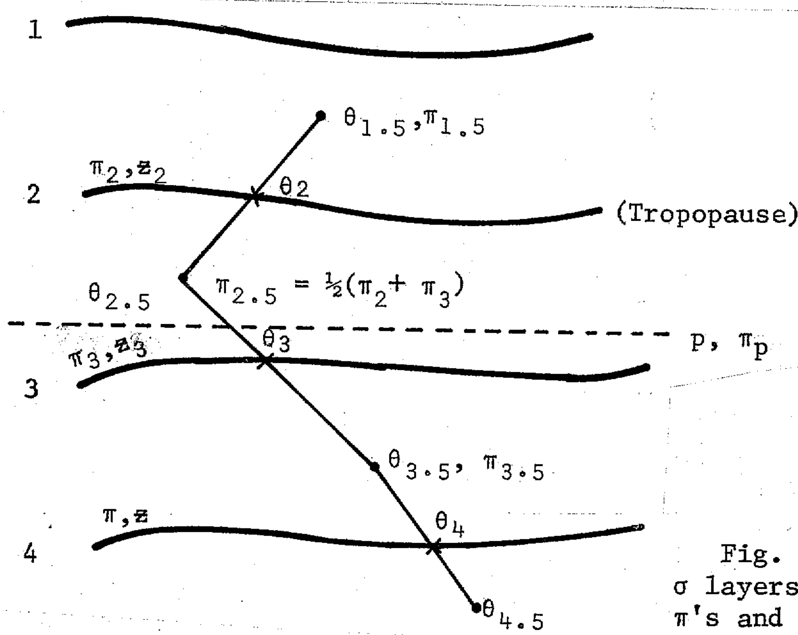

Fig. 1
σ layers, heights,
π's and temperatures

The solid lines are numbered σ surfaces, the dashed line is the mandatory p surface for which we want z_p.

The *old* procedure was, given a value of p and π_p, to search out the layer mean π values which bracket π_p ($\pi_{3.5}$ and $\pi_{2.5}$ in the particular situation of Fig. 1) then define, for the particular example,

$$b = \frac{\theta_{3.5} - \theta_{2.5}}{\pi_{3.5} - \pi_{2.5}}$$

$$a = \theta_{2.5};\ (\pi_o = \pi_{2.5})$$

and integrate from the σ level separating the layers ($z_\sigma = z_3$) to the pressure level. This integration could go up or down from σ level 3 depending upon whether p or π_p was less or greater than π_3, but the integration would not go beyond the center ($\overline{\pi}^\sigma$) of either layer. If the π_p were on the "other side" of the center of the layer, the integration would start from the next σ level up or down.

The *new* procedure is, given the same value of p and π_p, to search out the σ *level* π values which bracket π_p (π_2 and π_3 in this case). At those levels then we further define values of θ obtained by interpolation of θ, linear with π, from the adjacent layers. For example

$$\theta_2 = \frac{\theta_{1.5}(\pi_{2.5} - \pi_2) + \theta_{2.5}(\pi_2 - \pi_{1.5})}{\pi_{2.5} - \pi_{1.5}}$$

Then the slope of the θ-π line is

$$b = \frac{\theta_3 - \theta_2}{\pi_3 - \pi_2}$$

while

$$a = \theta_{2.5} \qquad (\pi_o = \pi_{2.5})$$

for the particular example. The integration then proceeds from $z_\sigma = z_3$ up to the desired pressure level. In this case, the integration could go beyond the middle of the layer but does not go past the next σ *level*.

The only difference between the two procedures is in the specification of the slope term b. And indeed through most of the atmosphere (anywhere below level 3 or above 2 in Fig. 1) the old vs. new values give all but identical results for b and thus for the calculated heights. They differ by no more than a meter or two as shown by tests.

However, it is in the layer under the tropopause, where a large change in the lapse rate takes place, that the old procedure causes difficulties. In effect, the old procedure causes (in Fig. 1) z_p to be larger than the value calculated by the new. This is not difficult to see as the old procedure integrates along the θ_3 to $\theta_{2.5}$ line while the new would integrate along a line parallel to the θ_3 to θ_2 line but *drawn through* $\theta_{2.5}$. The old method is thus warmer than the new, causing the difference in z_p.

Similarly, if the desired pressure level lay in the upper half of the same layer a similar thing happens but with an important reversal. The old method, being warmer, gives a *lower* value to z_p (we are integrating down now instead of up) than does the new.

If, as is frequently the case, two mandatory levels fall in the above positions--one in the upper half, one in the lower half of the layer under the tropopause--the old procedure produces a mandatory layer thickness temperature considerably colder than the new. Frequently, this thickness temperature is actually substantially unstable (by as much as 20°K) with respect to the mandatory layer below while the new method thickness and its associated temperature gives no trouble at all. This is obviously a correction to an error.

Also, it is possible to show that if the π_p level happened to coincide with $\pi_{2.5}$ you would get *different* values of z_p depending upon whether you integrated down from level 2 or up from 3 using the old procedure but you would get the same correct value for z_p using the new.

For these reasons, we are replacing the old procedure with the new in all NMC PE models--the 6L coarse mesh, the LFM and the 8L GLOBAL and Hemispheric models.