U.S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL WEATHER SERVICE
NATIONAL CENTERS FOR ENVIRONMENTAL PREDICTION

OFFICE NOTE 415a

AN INEXPENSIVE TECHNIQUE FOR USING PAST FORECAST ERRORS TO IMPROVE FUTURE FORECAST SKILL: PART (I) ---- ADJOINT METHOD

ZHAO-XIA PU
UCAR VISITING SCIENTIST PROGRAM

EUGENIA KALNAY, JOHN C. DERBER, JOSEPH SELA
ENVIRONMENTAL MODELING CENTER

MAY 1995

---

* To be submitted to *Quarterly Journal of Roy Meteorological Society*

ABSTRACT

A simple, relatively inexpensive technique has been developed for using past forecast errors to improve the future forecast skill. The method uses the forecast model and its adjoint and can be considered as a simplified 4-dimensional variational (4-D VAR) system. One or two-day forecast errors are used to calculate a small perturbation (sensitivity perturbation) to the analyses that minimizes the forecast error. As in Rabier et al (1994) the longer forecasts started from the corrected initial conditions, although better than the original forecasts, are still significantly worse than the shorter forecasts started from the latest analysis, even though they both had access to information covering the same period. The reasons for the poorer performance of the forecast sensitivity method, which, in principle, is similar to a 4-D VAR data assimilation approach, are: a) the use of the model as a strong constraint in the correction of the initial conditions, which becomes a poorer assumption the longer the period considered, especially when the linear tangent model and adjoint include only minimal physics; b) the use of a single iteration in the forecast sensitivity approach, since the correction to the initial conditions has clearly not converged after one iteration; and c) the use of analyses rather than the observations in the definition of the cost function.

As a much less expensive alternative to 4-D VAR, we use the adjusted initial conditions from one or two days ago as a starting point for a second iteration of the regular NCEP analysis and forecast cycle until the present time (t=0) analysis is reached. Forecast experiments indicate that the new analyses result in improvements to medium-range forecast skill, and suggest that the technique can be used in operations, since it increases the cost of the regular analysis cycle by a maximum factor of about 4 to 8, depending on the length of the analysis cycle that is repeated.

The model used in these experiments is the NCEP's operational global spectral model with 62 waves triangular truncation and 28 σ- vertical levels. An adiabatic version of the adjoint (Navon et al 1992) was modified to make it more consistent with the complete forecast model, including only a few simple physical parameterizations (horizontal diffusion and vertical mixing as in Buizza 1992). This adjoint model was used to compute the gradient of the forecast error with respect to initial conditions.

KEYWORDS: Adjoint model, Data assimilation, Numerical Weather Prediction, Forecast errors, Sensitivity

1. INTRODUCTION

The accurate specification of initial conditions is vital for Numerical Weather Prediction (NWP). Many poor forecasts can be traced to errors in the initial conditions. Recently, considerable attention has been focused on the 3-dimensional (3-D) and 4-dimensional (4-D) variational assimilation methods (Talagrand 1987; Derber and Parrish, 1991; Navon et al. 1992; Courtier et al. 1993) to improve the initial conditions. While the 3-D techniques have become feasible (Derber et al, 1991), the 4-D variational techniques remain expensive and still on the edge of feasibility for operational implementation. In this paper we describe a simple, less expensive alternative to the 4-D variational system that can result in improvements to the forecast and data assimilation system.

It is well known that errors originating from uncertainties in the initial conditions grow with forecast time. Lacarra and Talagrand (1988) and Vukicevic (1991) investigated the short range evolution of small perturbations, and showed that the evolution of small errors in a nonlinear model can be approximated with a linear tangent model for about two days. At the US National Centers for Environmental Prediction (NCEP/NWS/NOAA, formerly National Meteorological Center) there has been some experimentation to remove the short-term systematic errors and initial growing errors. Thiebaux and Morone (1990) investigated the spatial and temporal statistical coherencies in the departures of short-range-forecasts from observed and analyzed geopotential field, and found that they can be used diagnostically as the basis for adjustment of forecast products: estimate the "stray" of the forecast cycle using the most recent history discrepancies between forecast and analysis, then remove the stray from current forecast. Their experiments demonstrated that this statistical stray-corrected method had a significant impact on forecast accuracy, indicating that it may be possible to use the systematic statistical character of past forecast error to improve the current forecast. Kalnay and Toth (1994) used the growing modes obtained in the NCEP "breeding cycle" to partially remove the growing errors in the analysis cycle through adjustment the first guess (initial condition), and their results indicated that the procedure has the potential of improving the first guess and subsequent forecasts. In this study, we attempt to use directly short range forecast error to adjust the initial condition through a 4-dimensional minimization process by the adjoint method, and the experiments will show how we could use the adjusted initial condition to improve the *future* forecast skill.

In the last decade, adjoint models have been applied in four-dimensional data assimilation (Le Dimet and Talagrand 1986; Derber,1987 and 1989,Thepaut and Courtier 1991, M. Zupanski,

1993, D. Zupanski, 1993, D. Zupanski and Mesinger, 1995), in optimal parameter estimation (Zou et al. 1992), in the determination of singular vectors (Buizza, 1993), as well as in forecast sensitivity studies (Vukicevic, 1992, Rabier et al, 1994, M. Zupanski 1995). One of the main advantages of adjoint model is that it combines the dynamics of the forecast model and observed (or analyzed) information in one system, so that the gradient of cost-function with respect to all initial parameters, (initial field and model input parameters) can be calculated with a single integration. If the cost function is defined as the distance between a forecast and verifying fields, then by adjusting the initial conditions, the difference between the adjusted solution and the original solution will give an estimate of the distribution of the initial error which will explain a part of causes of forecast error. This idea was developed and applied by Rabier et al. (1993, 1994) using the European Center for Medium-Range Weather Forecasts (ECMWF) model. They calculated the perturbation to the two-day-old initial conditions that would minimize the two-day forecast error over Northern Hemisphere, and compared the resulting forecast with the original two-days old operational forecast. The result showed a very substantial improvement of several poor forecasts, although in most cases the updated forecasts from two days earlier were not better than those launched from the latest initial conditions. Zupanski (1995) found that the forecast sensitivity improved when several iterations were performed.

In this paper, we present a simple extension of the ECMWF technique which uses the known error from past forecasts to improve future forecasts. The initial conditions of forecasts 1 or 2 days old is altered based on their observed forecast error, as done by Rabier et al. (1993). The altered fields are then used as a starting point for a second iteration of the regular NCEP analysis cycle in order to improve *future* forecasts. The experiments are performed with the NCEP model with a triangular truncation of 62 waves and 28 vertical levels. The mathematical formulation is described in section 2. The adjoint model (section 3) contains the complete dynamics of the model, but only a few simple physical parameterizations (horizontal diffusion and vertical mixing), as in Buizza (1993). In section 4, we present forecast sensitivity experiments similar to those performed at ECMWF (Rabier et al, 1994). The extension of the method to improve future forecasts technique by performing a new assimilation cycle, and the corresponding results are presented in section 5. Section 6 includes a summary and conclusions.

2. MATHEMATICAL FORMULATION

At any forecast time t, for a forecast variable X, the perceived forecast error δx is given by difference between forecast (X_t) and analysis(X_t^{ref})

$$\delta x = X_t - X_t^{ref} \qquad (2.1)$$

where $X_t = M(X(t_0))$, M is a forecast model, $X(t_0)$ is initial condition corresponding to X_t. A measure of the error over the forecast period is defined as the objective function **J,** based on a total energy norm, given by:

$$J = \|\delta X\|^2 = \frac{1}{2}\int_0^1 \iint_\Sigma (\nabla\Delta^{-1}\zeta\nabla\Delta^{-1}\zeta + \nabla\Delta^{-1}D\nabla\Delta^{-1}D + R_a T_r (\ln\Pi)^2 + \frac{C_p}{T_r}T^2) d\Sigma (\frac{\partial P_r}{\partial z}) d\eta = \frac{1}{2}\delta X^T W \delta X$$

(2.2)

where, $\delta X = (\zeta, D, T, \Pi)$ denote differences between the forecast and analyses for vorticity, divergence, temperature and surface pressure respectively. T_r, P_r is the reference temperature and pressure (Rabier et al., 1994), R_a is the gas constant for dry air, C_p is specific heat at constant pressure for dry air. Σ represents the integration domain. W is the matrix of weights defining the norm. The weights are a function of T_r, P_r R_a, and C_p. η is the vertical coordinate.

We would like to find an initial perturbation $\delta x(t_0)$ that minimizes forecast error as measured by the objective function. To minimize the objective function, most algorithms require the gradient of the objective function with respect to the initial conditions. Talagrand (1986) showed that the gradient is given by:

$$\nabla_{x(t_0)} J = \sum_{i=1}^{N} L^T W \delta x(t) \qquad (2.3)$$

where L is the linearization of the forecast model around the current solution and L^T denotes the adjoint of the operator **L.**

In the first iteration of most minimization algorithms, the correction to the initial conditions is equal to the gradient times a reasonable step size ρ. From this we obtain the new initial conditions:

$$X(t_0)|_{new} = X(t_0) - \rho \nabla_{X(t_0)} J \qquad (2.4).$$

In much of the following we take advantage of the fact that this single iteration of the minimization algorithm explains a substantial portion of the forecast error. The considerable cost of each iteration makes it impractical to perform the minimization with many iterations as recommended by Zupanski (1995). Examples on the impact of the use of several iterations will be shown in section 4.

3. MODIFICATIONS TO THE ADJOINT OF THE NCEP GLOBAL SPECTRAL MODEL

As can be seen in the above derivation, the adjoint model plays a vital role in the calculation of the gradient of the cost-function with respect to the initial conditions. To ensure that we obtain a reasonably accurate gradient, it is necessary for the adjoint model to be as consistent as possible with the forecast model. It is well known that the inclusion of all the model physics into the linear tangent model and its adjoint is both difficult and computationally expensive, although the results appear to improve with the inclusion of some physical parameterizations (Zupanski and Mesinger, 1995; Zou et al. 1993). Because of these difficulties, however, most applications so far have used simplified adjoint models. Errico et al. (1993) performed experiments with an adiabatic adjoint model which was able to accurately estimate the perturbation tendencies of a full physics model. Buizza (1993) found that by using a linear tangent adiabatic model with surface drag, horizontal diffusion and a simple vertical diffusion, he could derive linear perturbations that were reasonably close to the nonlinear evolution of a perturbation, thus validating also the corresponding singular vectors. At NCEP, an adiabatic adjoint for the global spectral model was developed by Navon et al. (1992), including a simple surface drag scheme and horizontal diffusion. Experiments showed that this version of the adjoint worked well for most of the atmosphere within short periods, but unrealistically large growth rates were observed at low levels of the model (Szunyogh, personal communication, 1994), a problem attributed to inadequacies in the model physics. Following Buizza (1993) and Thone (1986), we introduced, in addition to the surface drag and the horizontal diffusion, a linear vertical diffusion scheme. To assess the effect of the vertical diffusion parameteriztion on the spurious growth, the nonlinear NCEP global spectral model was integrated for 24 hours from 0000UTC 16 May 1985 initial conditions, but using different subsets of physical parameterizations. The results were

compared with those obtained running the operational model with full physics. Fig. 1 shows the kinetic energy norm of the different 24-hour forecasts at all model levels. In the run with no physics it is apparent that there is an unrealistically large increase of kinetic energy at low levels when compared with the run with full physics. The introduction of surface drag only improves the lowest model level, but the kinetic energy in levels 2-7 remains too large. The introduction of the new vertical diffusion and horizontal diffusion results in a solution much closer to the operational T62 model with full physics.

To further examine the approximation to the complete vertical diffusion parameterization, a small amplitude initial perturbation field δx_0 was introduced into the nonlinear model integration with the full physics package, and the difference between the two nonlinear integrations (staring from perturbed and unperturbed initial condition at 0000UTC 16 May 1985) was compared with a linear model integration which included the new linear vertical diffusion and the same perturbation (δx_0) as initial condition. Fig. 2 shows the comparison between the linear and nonlinear evolution for the temperature perturbation field at sigma level 13 (about 500mb) after 24 hours of integration. The agreement between the perturbation fields obtained from the linear and the nonlinear integration is generally good, containing small differences in phase but reproducing all the main features. This suggests that, in agreement with previous investigators, the dry adiabatic linear model with surface friction and vertical diffusion can reproduce fairly well the nonlinear perturbations of the model with full physics for short range forecasts. This new vertical diffusion scheme was included in the linear and adjoint models used in this paper.

4. SENSITIVITY OF SHORT RANGE FORECAST ERRORS TO PERTURBATIONS IN THE INITIAL CONDITIONS

(a) Experiments

In this section, sensitivity experiments similar to those performed at ECMWF (Rabier et al., 1994) are performed using the NCEP adjoint system. Again, 0000UTC 16 May 1985, available from the NCEP/NCAR Reanalysis Project, was arbitrarily chosen as the initial condition. The current NCEP operational data assimilation system (as of January 1995) was used to create the analysis on a 6-hour cycle. This system uses a 3-dimensional variational analysis scheme developed by Derber and Parrish in 1991 and which has undergone further improvements.

In order to test the sensitivity of the one or two-day forecast error to changes in the initial conditions, three sensitivity experiments are performed and compared. In Experiment 1 we use a one-day forecast error, where the forecast error is defined as the difference between the forecast and the analysis at 0000UTC 17 May 1985; in Experiment 2 we use a two-day forecast error, similarly defined at 0000UTC 18 May 1985; and in Experiment 3, we also use a two-day forecast, but the forecast error in the cost function is defined as the sum of all the forecast differences with the analyses available every six hours within the period 0000UTC 16 May 1985 through 0000UTC 18 May 1985. A conjugate gradient method (Gill 1989) was employed in our system, with an optimal step size defined as in Derber (1987).

Fig. 3 illustrates the vertical distribution of the total energy norm of the forecast error before and after a single iteration in Experiments 1 and 2, showing that the error decreases significantly at every sigma layer. The effect of performing multiple iterations is shown in Fig. 4 for Experiment 3. As in Zupanski (1995) we find that although the first iteration reduces substantially the cost function (by 18%), additional iterations continue to decrease it, with a total reduction of about 45% after 10 iterations (Fig. 4a). Similarly, Fig. 4b shows that the forecast error throughout the two-day forecasts after one iteration is reduced by about 20%, and after another 9 iterations by additional 25%. Note that in the extended period forecast error, we found that not only the 48-h forecast is improved, but the 3-5 day forecasts are also significantly better. There is a clear advantage of having more iterations, although one iteration is enough to improve the control forecast. Zupanski (1995), using a regional model and its adjoint, found that the sensitivity fields obtained after one and ten iterations were not very similar, with projections corresponding to angles of up to 50° between them. But we should also notice that the descent

of the cost-function in the first iteration is faster than for the other iterations.

Since the first iteration explains more forecast error than any of the following ones, and because of the large expense of performing many iterations of the descent algorithm, ECMWF has chosen to perform a single iteration in their experiments (Rabier et al, 1994), with a value of the step size ρ= 1/100, 1/150 or 1/200 (depending on the season) multiplied by the gradient, to generate the sensitivity perturbations. In this study, our main purpose is to use the improved initial conditions as the starting point for a second iteration of the regular NCEP analysis cycle (Section 5). We are interested in the potential operational application of this method, which limits the available computer resources, so that we have also chosen to perform only a single iteration assuming that such single iteration will result in a substantial reduction of the forecast error while remaining computationally efficient. Since the fixed step size used by Rabier et al. (1994) may not be sufficiently accurate, we determine the step size using the technique described in Derber (1987), with 1/100 or 1/200 as the guess step size ρ_g. The optimal step size ρ is given by:

$$\rho = \rho_g \frac{\sum_{i=1}^{N} W[(X_i - X_i^{ref})]^T X_g'(t_i)}{\sum_{i=1}^{N} [W X_g'(t_i)]^T X_g'(t_i)} \qquad (4.1)$$

where $X'_g(t_i)$ is the difference between the forecasts results from integrating the initial condition $X(t_0)$ and $X(t_0)+\rho_g \nabla_{x(to)} J$. To calculate the optimal step size for one iteration, the forecast model needs to be integrated one time from the initial condition $X(t_0)+\rho_g \nabla_{x(to)} J$.

(b) Impact of the sensitivity perturbations on medium range forecasts

In this subsection we compare the impacts on medium range forecasts of the three forecast sensitivity experiments discussed in the previous subsection, using a single iteration. We also compare a control forecast performed from an unperturbed analysis for 0000UTC 16 May 1985.

Fig. 5a shows the comparison of the total error energy norm of the three sensitivity forecasts with the control forecast. Figs. 5b, c, and d illustrate the global RMS forecast errors for

temperature, divergence and vorticity respectively. The results indicate that the one-day sensitivity perturbation (experiment 1) improves not only the first day, but that the benefit extends to days 2 through 4. The two-day sensitivity experiment (experiment 2) improvement extends throughout the whole forecast period (days 3 through 7). Comparing experiment 2 with experiment 3, in which the forecast error was defined throughout the two-day forecast and not just at the end of the two days, we observe that experiment 3 is clearly better during the first 4 days, but that at days 5 and 7 experiments 2 and 3 give similar errors. Note that experiment 3 results in considerable larger changes in the initial conditions than the other two experiments. Experiments indicated that the length of the period and the density of data insertion in the period affected the initial perturbation and subsequent forecasts. This effect will be further tested with the several possible operational configurations in subsection 5(b).

The results of these experiments agree with the ECMWF experience that the benefit of the perturbed initial conditions extends beyond the period for which the forecast sensitivity perturbation was initially calculated.

(c) Evolution of the sensitivity perturbation

A second experiment on forecast sensitivity on short and medium range forecasts was performed with initial conditions corresponding to 0000UTC 23 February 1995. The sensitivity was calculated from 2-day forecast errors at 0000UTC 25 February 1995, as in Experiment 2. Fig.6 shows the 500mb geopotential height difference between the sensitivity and the control forecast at 0, 24, 48, 72, 96, and 120 hours. We can see that the small initial differences (plotted with a 3-meter contour interval) lead to very large differences after 120 hours (plotted with a 25m contour interval). These large differences take place in both the Northern and the Southern Hemispheres. Table 1 presents the anomaly correlation scores verified against operational analyses for the control and the sensitivity forecasts, showing large improvements in both hemispheres. An inspection of the distribution of the initial perturbation of geopotential height at 500mb (corresponding to T+00 in Fig. 6) indicates that there may be two dominant error sources in the Northern Hemisphere, over Canada and the North Pacific areas. The five-day forecast differences are also maximum over North America in this case. A check of the corresponding 5-day forecast of the NCEP operational model (at T126/28 level resolution) indicates that the anomaly correlation scores on 0000UTC 28 February 1995 were also unusually low over North America, and within normal range for other areas of the Northern Hemisphere (P. Caplan, pers. comm. 1995). This confirms that the large error growth of the sensitivity

perturbation is present even in the high resolution model, and that the adjoint method is a useful tool to trace back growing initial perturbations associated with the forecast error.

5. USE OF PAST FORECAST ERRORS TO IMPROVE FUTURE FORECAST SKILL

(a) Comparison of the sensitivity forecasts with regular forecasts from latest initial conditions

So far, our results confirm the experience of Rabier et al. (1994), showing that the use of the short range forecast errors can improve substantially the original forecast, even beyond the period for which the error was computed. But the crucial question for an operational center like NCEP is whether we can use this procedure to improve the skill of *future* forecasts. In other words, if we use the observed two-day forecast error to improve the initial conditions from two days ago, will the improved 5-day forecast from two days ago be also better than the 3-day forecast from today? Unfortunately, as will be seen, the answer to this question is that, in most cases, this is not the case. This lack of improvement with respect to the latest forecast was also observed by Rabier et al. (1994). Therefore, taking advantage of our knowledge of the data available until the present time just to correct older initial conditions is not enough to make forecasts better than the regular forecasts made from the analysis cycle using all the data available until today. There may be at least three reasons for this result: a) the use of the model as a strong constraint within the forecast sensitivity procedure; b) the use of a single iteration to obtain the improved initial conditions; and c) the fact that the latest analysis uses the observations directly.

However, it seems natural that if we can improve the initial conditions from two days ago, it should be possible to perform a second iteration of the regular analysis cycle, *starting from the improved initial conditions from two days ago,* and that this iterated analysis could then lead to better forecasts from *today.*

We tested first this idea for 7 consecutive cases from 16 to 22 March 1995. At 00Z every day, we calculated the sensitivity perturbation from the two days forecast error, then compared the sensitivity 5-day forecasts with the original 5-day control forecasts. Fig. 7 shows the comparison of the 5-day forecast anomaly correlations scores verified against control analyses for 500mb geopotential height. Not surprisingly, the 5-day sensitivity forecasts (which take advantage of the information available for two additional days in the computation of the two-day

forecast error) are in most cases substantially better than the original 5-day forecasts, in both hemispheres. However, the 3-day forecasts from the regular analysis cycle, also taking advantage of the data gathered over the last two days, shows an even larger improvement (Fig. 7). A similar conclusion was reached using one-day sensitivity forecasts: we found that the 5-day sensitivity forecasts were much better than the 5-day control forecasts, but they did not exceed the skill of the corresponding 4-day forecasts (figure not shown).

As indicated above, we tried to solve the lack of improvement upon future forecasts by a simple extension of the Rabier et al.(1994) method, in which the sensitivity perturbation of short range forecasts is used as a starting point for a second iteration in the analysis and forecast cycle for the last two days. We called this an iterated analysis-forecast cycle.

(b) An iterated analysis-forecast cycle using the sensitivity perturbation

The initial conditions for the operational medium range global spectral model are provided by the NCEP spectral statistical interpolation (SSI) scheme, a 3-D variational analysis (Derber et. al., 1991). In the NCEP 6-hour analysis/forecast cycle, the 6-hour forecast from the global spectral model is used as a first guess for the SSI, which finds the 3-D analysis fields that best fit both the first guess and all the observational data gathered for the next 6-hour period.

In the iterated analysis cycle, we introduce the 2-day sensitivity perturbation into the 2-day-old analysis. We then repeat the analysis cycle for the last two days using the improved initial analysis until we reach the end of the sensitivity period. The verifications are made against the control analysis. Fig. 7 also shows the anomaly correlation of the 3-day forecast that starts from the new analysis, indicating that, for most cases, the new 3-day forecast is better than the original forecast. Fig. 8 shows the 5-day forecasts from the regular and iterated analysis cycle using the two-day sensitivity perturbation. It is encouraging that, in five out of the seven cases, and in both hemispheres, the new 5-day forecast is equal or better than the regular (operational) forecast.

The conclusion of this experiment is that the iterated cycle has the potential of improving the forecast skill, because it improves the corresponding 3 to 5-day operational forecasts that used the same available information. Similar results were obtained using a one-day sensitivity perturbation, and using it as starting point for a repeated one-day analysis cycle (not shown).

(c) Test of several possible operational configurations of the iterated analysis cycle

In this final test, we chose a period from March 18 to April 1, 1995. All tests were done in a configuration that could be implemented operationally, and the same procedure was applied every day. The iterated cycle was performed in a continuous fashion as follows: a) We use the 0000 UTC analysis to compute the sensitivity perturbation for the one or two-days previous analysis; b) We use the modified analysis to repeat the SSI analysis cycle until new current initial conditions are obtained; c) Medium range forecasts are performed and compared with the forecasts from the original (not iterated) analysis cycle.d) This analysis is then used to calculate the next perturbation and start the next analysis cycle.

Four possible configurations were tested: 1) A 24-hour forecast error was used in the one-day iterated assimilation period; 2) A 48-hour forecast error was used in the two-day iterated assimilation period; 3) As in 2), but 24, 36, and 48-hour forecast errors were used to determine the sensitivity perturbation; and 4) As in 2), but 36 and 48-hour forecast errors were used to determine the sensitivity perturbation. The individual forecast results corresponding to tests 1 and 2 are presented in Fig. 9, and those of tests 3 and 4 in Fig. 10. The average scores verified against the control analysis over the 15-days of experimental forecasts are presented in Table 2.

The iteration of the analysis cycle results in a small overall improvement with respect to the operational control, for both Northern and Southern Hemispheres, even though it is not successful in every case. Overall, the use of the two-day sensitivity results in more improvements than the use of a one-day sensitivity. This is not surprising, since the errors are better defined after two days for two reasons: a) The 2-day forecast errors are larger, and therefore, the perceived "forecast errors" are less affected by the unknown analysis errors; and b) The growing errors are better organized after two days than after one day.

With respect to the use of just the 48-hour forecast errors or adding the 24 and 36 hour errors in determining the sensitivity patterns, the results seem to favor slightly the use of only the 36 and 48 hour forecast errors in the Northern Hemisphere. This would be in agreement with the two arguments presented above: by including the 36-hour forecast errors (at a time in which the growing errors are already organized), we increase the signal-to-noise ratios of the cost function. In the Southern Hemisphere, which is less well observed, and where the uncertainties in the analysis are larger, the results suggest that it may be better to only include the 48 hour forecast errors. However, given that the difference between the one and the two-day iterated assimilation impact is not large, it may still be desirable to perform this procedure operationally with a one-day sensitivity approach.

Fig.11 shows the impact of the iterated cycle for a typical 500mb geopotential height 5-day forecast (0000UTC 29 March 1995) over the US area. The 5-day control forecast misses a low-pressure cyclone over Utah, but 5-day new forecast predicted this system much better, even though it is still slightly too far to the south and east.

6. SUMMARY AND CONCLUSIONS

In this study, we presented a simple, relatively inexpensive technique for using past forecast errors to improve the future forecast skill. The method is an extension of the forecast sensitivity studies of Rabier et al (1994), and can be considered as a very simplified approach to 4-dimensional variational data assimilation. The sensitivity patterns are obtained by minimizing a cost-function defined by the one or two-day forecast error norm. Since we are interested in a cost-effective approach, we assume that the first iteration of the minimization algorithm will be enough to achieve a substantial reduction of the forecast error, but not necessarily a true indication of the analysis errors. We then use the perturbed one and two-day-old analysis as an improved starting point for a repetition of the regular analysis cycle until the current initial conditions are reached. Forecasts from these iterated analyses are in general better than the original forecasts.

We first tested the NCEP adjoint system by first performing experiments similar to those of Rabier et al (1994). We found that it was necessary to introduce a simplified vertical diffusion scheme into the linear and adjoint NCEP adiabatic models developed by Navon et al (1992), as done by Buizza (1993), in order to avoid spurious increases of energy in the lower layers of the model.

The sensitivity results obtained with this system are similar to those of Rabier et al (1994): The 5-day forecasts from the 2-day-old analyses perturbed by the forecast sensitivity pattern (obtained from the 2-day forecast errors) were substantially better than the original 5-day forecasts. However, this important result, which proves that the use of the adjoint model is a powerful tool to estimate errors in the initial conditions, does not by itself lead to improved operational forecasts: When we compared the improved 5-day forecasts from 2-days old initial conditions with the 3-day forecasts from the latest available analyses, the latter were still considerably better, also in agreement with Rabier et al (1994).

Since the forecast sensitivity method is, in principle, similar to a 4-D variational data assimilation approach, it is important to consider the question why the 5-day forecasts with initial conditions corrected using the perceived 2-day errors are so much worse than the 3-day forecasts from the latest analysis, since both have access to information covering the same period. There are several possible reasons for this: One is the use of the model as a strong constraint in the correction of the initial conditions, which becomes a poorer assumption the longer the period considered, especially with a linear tangent model including minimal physics. A second reason is the use of a single iteration in the forecast sensitivity approach, since the correction to the initial conditions has clearly not converged after one iteration. A third reason is the use of analyses rather than the observations in the definition of the cost function. By contrast, in the NCEP regional 4-D VAR approach, which includes an estimation of the background errors, most of the model physics in the linear tangent and the adjoint models, direct assimilation of the data, and about ten iterations, the forecasts are almost without exception better than the forecasts from the control Optimal Interpolation analysis (Zupanski and Zupanski, 1995). Thus the 4-D VAR approach is attractive but computationally rather expensive.

As a less expensive alternative to the full 4-D VAR approach, we used the improved initial conditions from the sensitivity pattern to restart an analysis cycle for the last two days. In this case, in contrast to the sensitivity approach, most of the forecasts from the iterated analyses were equally or more skillful than the original forecasts. We found that the use of 2-day sensitivity perturbations produced overall slightly better results than the use of one-day sensitivity patterns. This is probably due to the fact that the one or two-day "forecast errors," obtained as the difference between the forecast and the analysis, are only perceived errors, and are affected by unknown errors in the analysis used for verification. Therefore, the signal-to-noise ratio of the perceived forecast errors is better after two days, simply because the forecast errors are larger. Moreover, the growing errors that we are trying to reduce from the initial conditions become better organized and dominant over two days than over one day. We also found that, at least over the Northern Hemisphere, the use of the 36-hour forecast errors in addition to the 48 hour errors within the cost function led to somewhat better results, presumably because it also improved the signal-to-noise ratio. In the Southern Hemisphere, where the analysis uncertainties are larger, the use of only 48-hour errors in the cost function was slightly better.

In our system we have included an error norm defined globally. It is possible that defining the error norm only over the Northern Hemisphere extratropics, as in Rabier et al (1994) would lead to even larger improvements for this area (but to no improvements in the Southern

Hemisphere operational forecasts).

The new scheme requires only one iteration of the minimization algorithm per day, as well as an iteration of the analysis cycle for one or two days. Depending on whether one or two-day errors are considered in this procedure, this increases the cost of the analysis cycle by a factor of about 4 to 8, (compared to a much larger increase with full 4-D variational systems) and therefore becomes computationally feasible with present day systems. It is possible that additional reduction in the cost of this system (and the 4-D variational system) can be made by including additional approximations (e.g., lower resolution, etc.). We plan to further test this system for possible operational implementation at NCEP.

Acknowledgments

We would like to thank Drs. M. Zupanski, D. Zupanski, W.-S. Wu, Z. Toth and S.-Y. Hong for very helpful discussions. The code to calculate the total energy norm was kindly provided by Dr. Istvan Szunyogh. Mr. Yuejian Zhu provided all the data for this research. Drs. Dusanka Zupanski, Zoltan Toth and Jean Thiebaux made helpful suggestions on the manuscript. The first author is supported by the UCAR/NCEP Visiting Scientist Program.

REFERENCES

Anderson, E., Pailleux, J., Thepaut, J.-N., Eyre J. R., McNally, A. P., Kelly, G. A. and Courtier, P., 1993: Use of cloud-cleared radiances in three/four-dimension variational data assimilation, *Q.J.R., Meteorol.Soc*, **120**,627-653

Barkmeijer, J., 1992: Local error growth in a barotropic model, *Tellus*, **44A**,314-323.

Buizza, R., 1994: Sensitivity of optimal unstable structures. *Q. J.Roy Meteorol. Soc.*, **120**,429-451.

Buizza, R., Tribbia J., Molteni, F. and Palmer, T., 1993: Computation of optimal unstable structure for a numerical weather prediction model. *Tellus*, **45A**, 388-407.

Buizza, R., 1992: Impact of a simple vertical diffusion scheme and of the optimization time interval on optimal unstable structure. *ECMWF Technical Memorandum.* No.192.

Derber, J. C., 1989: A Variational continuous assimilation technique, *Mon. Wea. Rev.*, **117**, 2437-2446.

Derber, J. C., 1987: Variational Four-Dimensional Analysis Using Quasi-geostrophic Constraints. *Mon.Wea.Rev.*, **115**. 998-1008.

Derber, J. C., Parrish, D. F., Lord, S. J., 1991: The new global operational analysis system at the national meteorological center. *Weather and Forecasting*, **6**,538-547.

Derber, J. C., Parrish, D. F., Wu, W.-S., Pu, Z.-X. and Rizvi, S. R. H., 1994: Improvements to the operational SSI global analysis system. 10th conference on numerical weather prediction. July 18-22,1994, Portland, AMS.

Errico, R. M. 1993, T. Vukicevic, and Raeder, K., Examination of the accuracy of a tangent linear model, *Tellus*, **45A**,462-477.

Errico, R. M. and T. Vukicevic, 1992: Sensitivity analysis using an adjoint of the PSU-NCAR mesoscale model. *Mon. Wea. Rev.*, **117**,2437-2446.

Gill, P. E., Murray, W. and Wright M. H., 1981: *Practical optimization.* Academic Press, London, 401 pp.

Kalnay, E., and Toth, Z., 1994: Removing growing errors in the analysis.10th conference on numerical weather prediction, July 18-22,1994,Potland, AMS, 212-215.

Lacarra, J. F. and Talagrand, O., 1988: Short range evolution of small perturbation in a barotropic model. *Tellus*, **40A**,81-95.

LeDimet, F. X., and Talagrand, O., 1986: Variational algorithms for analysis and assimilation of meteorological observations: Theoretical aspects. *Tellus*, **38A**, 91-110.

Navon.I.M., Zou, X., Derber, J. C. and Sela, J. G., 1992: Variational data assimilation with

an adiabatic version of the NCEP spectral model. *Mon. Wea. Rev.*, **115**, 1479-1502.

Rabier, F., Courtier, P., Pailleux, J., Talagrand, O., and Deasko, V., 1993: A comparison between four-dimensional variational assimilation and simplified sequential assimilation relying on three-dimensional variational analysis *Q.J.R.Meteorol.Soc.* , **119**, 845-880.

Rabier, F., Klinker, E., Courtier, P. and Hollingsworth, A., 1994: Sensitivity of two-day forecast errors over the Northern Hemisphere to initial conditions. *Tech.Memo.203*, ECMWF, Reading, U.K., 38pp

Szunyogh, I. , Kalnay, E. and Toth, Z., 1995: A comparison of lyapunov vectors and optimal vectors in a low resolution GCM. *Submitted to Tellus.*

Talagrand, O. and Courtier, P., 1987: Variational assimilation of meteorological observations with the adjoint vorticity equation----part I.Theory, *Q.J.Roy.Meteor.Soc.*, **113**,1311-1328.

Thiebaux, H. J., Morone, L. L., 1990: Short-term systematic error in global forecasts: their estimation and removal. *Tellus*, **42A**, 209-229

Thone, I. and Mahrt, L., 1986: A simple model of the atmospheric boundary layer: sensitivity to surface evaporation, *Bound. Layer. Meteor.*, **37**,129-148.

Vukicevic, T., 1991: Nonlinear and linear evolution of initial forecast error. *Mon.Wea.Rev.*, **119**,1602-1611

Zou, X., Navon, I. M. and Sela, J. G., 1993: Variational data assimilation with moist threshold processes using the NMC spectral model. *Tellus*, **45A**, 370-387.

Zou, X., Navon, I. M. and LeDimet, F. X., 1992:An optimal nudging data assimilation scheme using parameter estimation. *Q.,J.,Roy.Meteorol.Soc.*, **118,**1163-1186.

Zupanski, M., 1993, Regional four-dimensional variational data assimilation in a quasi-operational forecasting environment. *Mon.Wea.Rev.*, **121**,2396-2408

Zupanski, M., 1995,An iterative approximation to the sensitivity in calculus of variations, *Mon.Wea.Rev.,*in press.

Zupanski, D., 1993, The effects of discontinuities in the Betts-Miller cumulus convection scheme on four-dimensional variational data assimilation. *Tellus*, **45A**,511-524

Zupanski, D., Mesinger, F.,1995, Four-dimensional variational assimilation of precipitation data. *Mon.Wea.Rev.*, **123**, 1112-1127.

Zupanski, M., and Zupanski, D., 1995: Recent Developments of the NMC's regional 4_D variational data assimilation system. Proceedings of the Second International Symposium on Assimilation of Observations in Meteorology and Oceanography, Tokyo, Japan,13-17 March 1995, pp 367-372.

Figure Captions

Fig.1 Vertical cross section of kinetic energy norms after 24 hours integration of the T62 global spectral model with no physics, with surface drag, with surface drag and simple vertical diffusion, and with the full physics (operational model).

Fig.2 The linear (2a) and nonlinear (2b)evolution of a perturbation δX_0 after 24 hours. Dashed lines represent negative values.

Fig.3 Vertical cross section of the total energy norm of forecast error before (solid curve) and after (dashed curve) one iteration.

Fig.4a Variation of the normalized cost-function (sum of total error norms) with the number of iterations.

Fig.4b The total energy norms at every 24 hours in two days and extended forecast period. Solid curve for the error norm of first guesses (control forecast) and long dash line for an error norm after 1 iteration and short-dash line for after 10 iteration.

Fig.5a The forecast error measured by total energy norm.

Fig.5b The root mean square forecast error of the temperature

Fig.5c The root mean square forecast error of the divergence

Fig.5d The root mean square forecast error of the vorticity

Fig.6 Height difference between the sensitivity and the control forecasting for steps 0-120 hours from 0000UTC 23 February 1995.

Fig.7 The anomaly correlation scores for 500mb geopotential heights, for 5-day sensitivity forecast, corresponding 3-day control (operational) forecast, 5-day control forecast and for the 3-day forecast from the new cycle (iterated). Dates (March of 1995) on the horizontal axis denote the starting dates of forecasts.

Fig.8 Same as Fig.7, except 5-day control (operational) forecast and 5-day forecast from the new cycle (iterated).

Fig.9 Comparison of 5-day forecast anomaly correlations scores verified against control analyses for 500mb geopotential height. from control forecast, test 1 and test 2. Dates (March of 1995) on the horizontal axis denote the starting dates of forecasts.

Fig.10 Same as Fig.9, except test 3 and test 4.

Fig.11 5-day control forecast (a), 5-day forecast in iterated cycle(b) and analysis (c) of 500mb geopotential height field over the US area at 0000UTC 29 March 1995.

Table Captions

Table.1 The anomaly correlation scores of 1-5-day forecast from 0000UTC 23 February 1995 for 500mb geopotential height

Table.2 Comparison of the average anomaly correlation scores of 1-5-day forecast for 500mb geopotential height. Starting dates of the forecasts ranged from 18 March 1995 to 1 April 1995

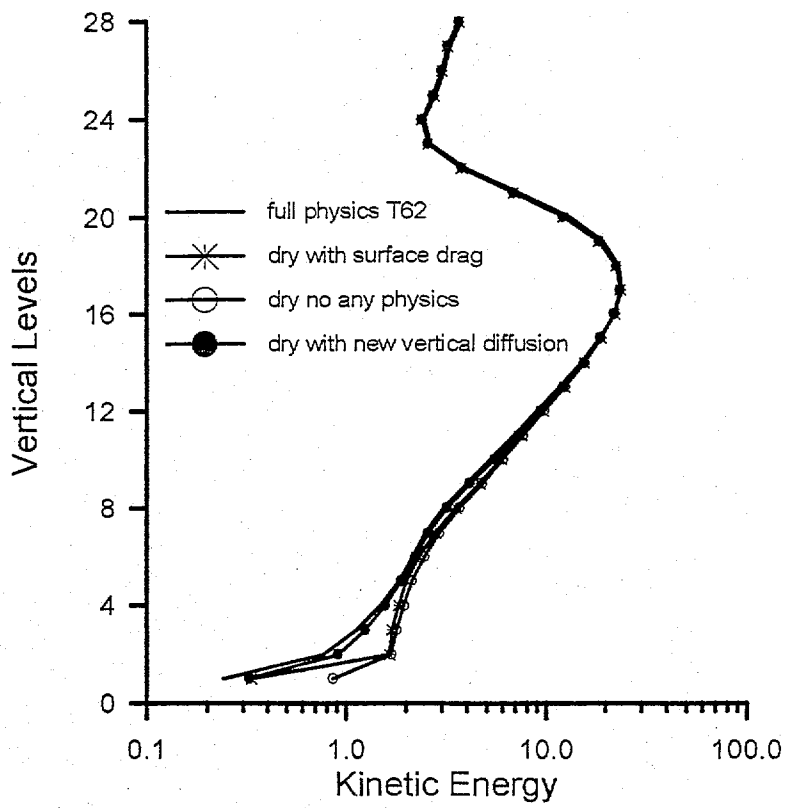

28
24
20
16
12
8
4
0
Vertical Levels
full physics T62
dry with surface drag
dry no any physics
dry with new vertical diffusion
0.1
1.0
10.0
100.0
Kinetic Energy

Fig. 1

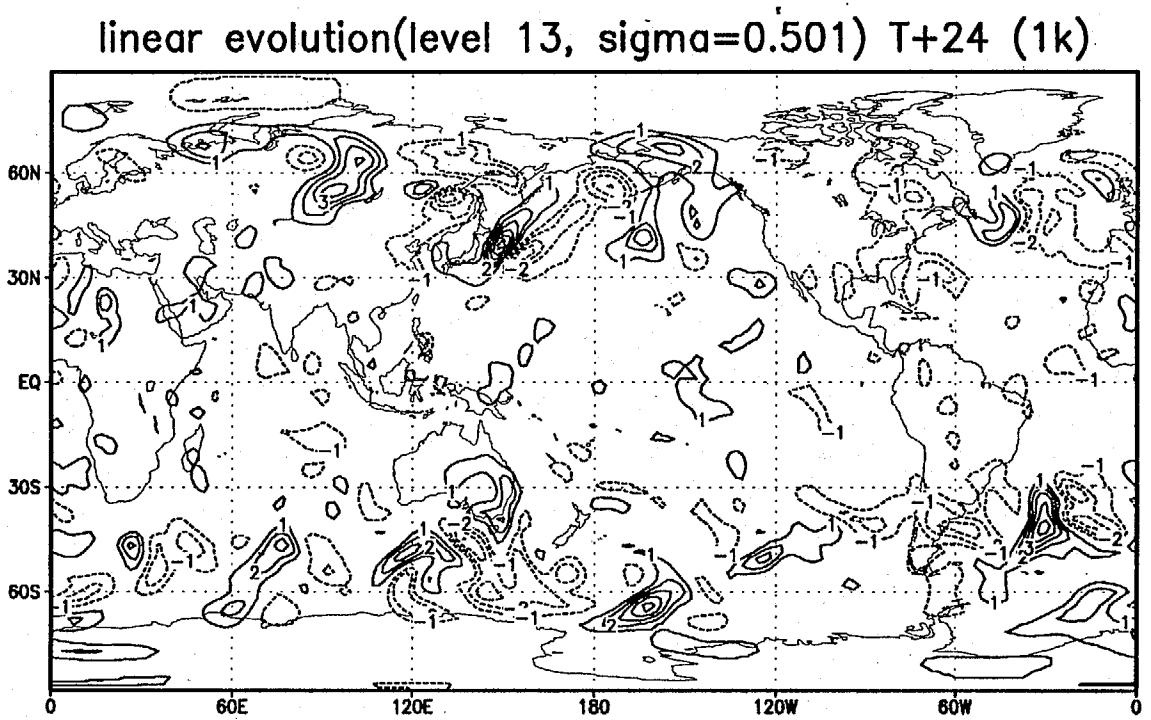

linear evolution(level 13, sigma=0.501) T+24 (1k)
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W
0

Fig. 2a

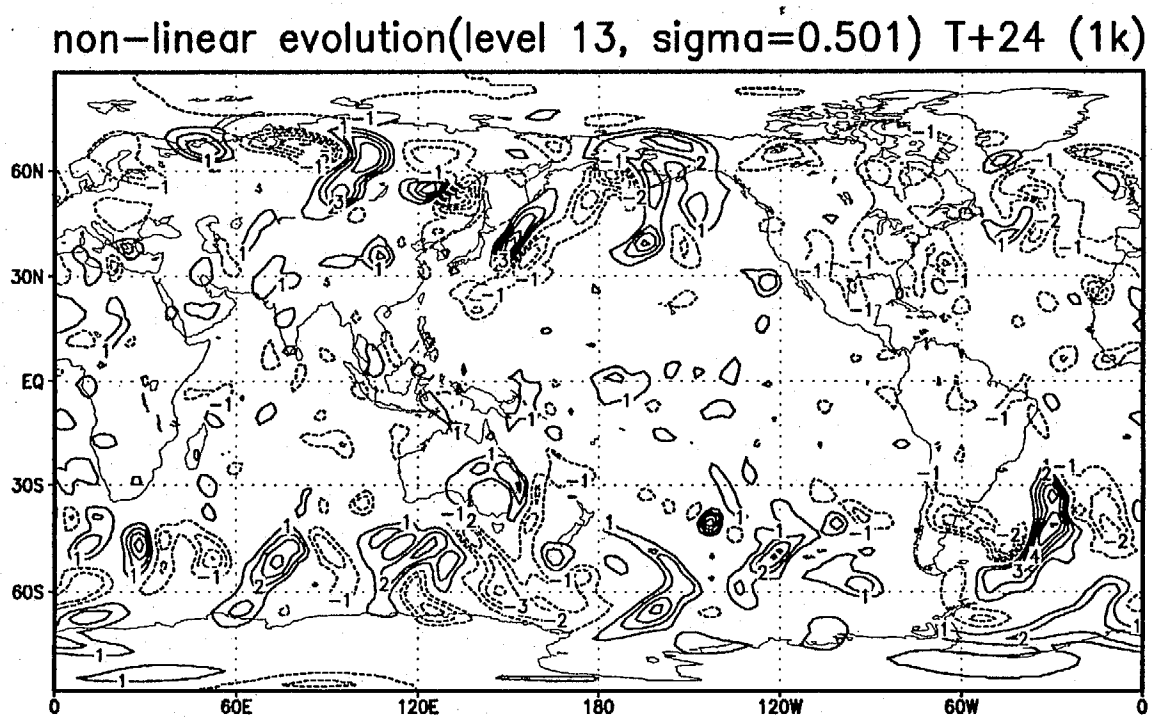
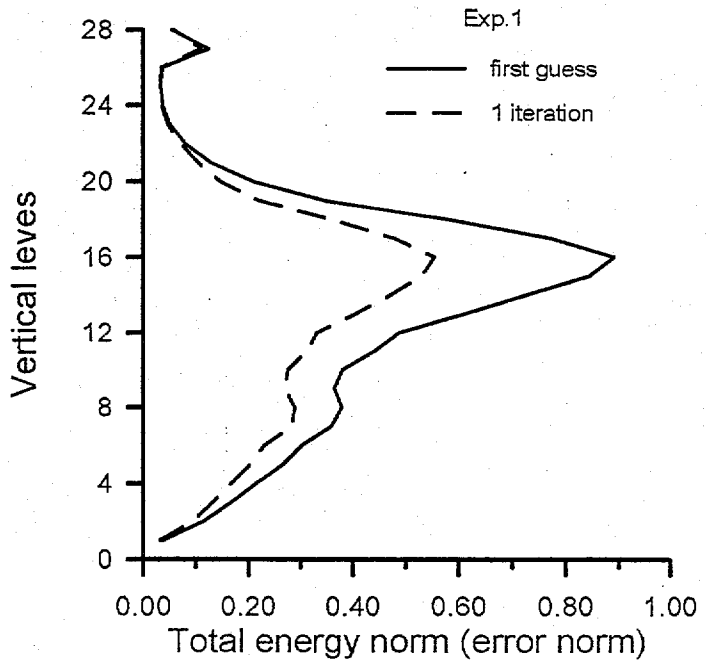

non-linear evolution(level 13, sigma=0.501) T+24 (1k)
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W
0

Fig. 2b


Exp.1
first guess
1 iteration
Vertical leves
28
24
20
16
12
8
4
0
0.00
0.20
0.40
0.60
0.80
1.00
Total energy norm (error norm)

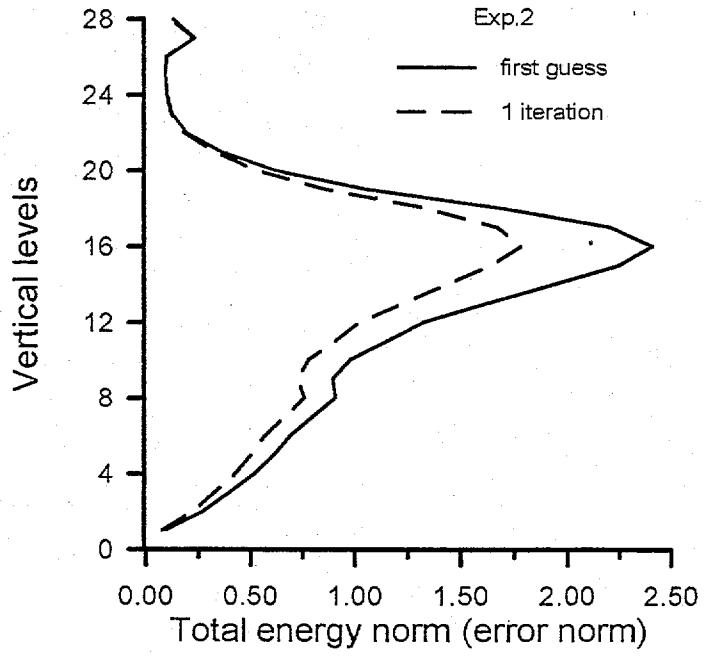

Exp.2
first guess
1 iteration
Vertical levels
28
24
20
16
12
8
4
0
0.00
0.50
1.00
1.50
2.00
2.50
Total energy norm (error norm)

Fig. 3

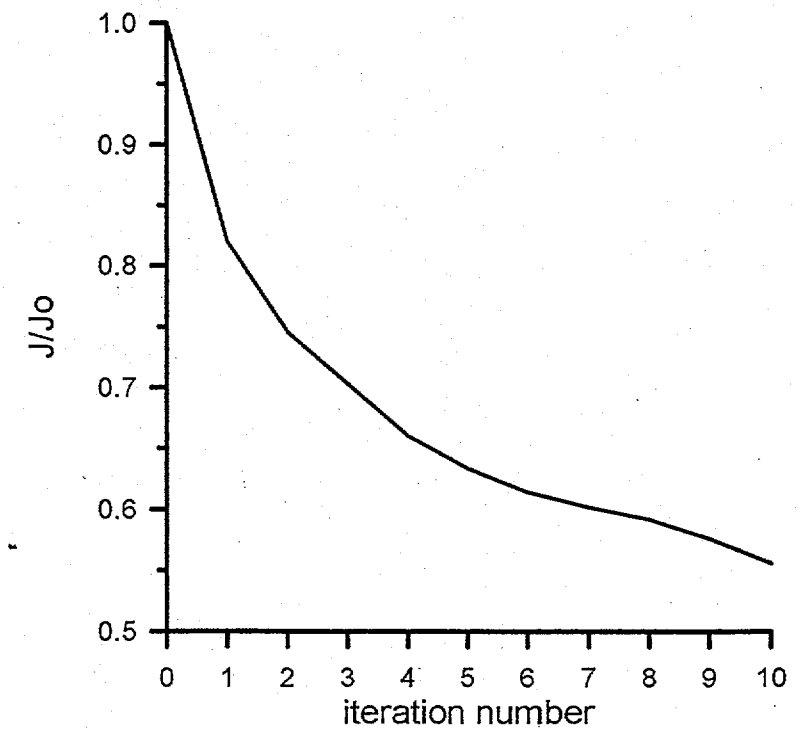


Fig. 4a

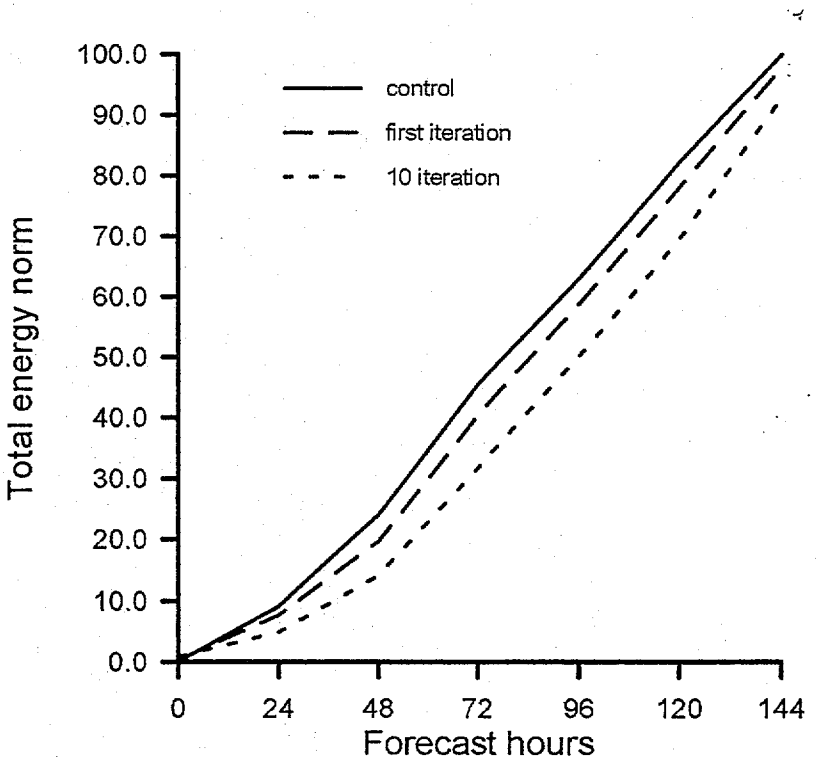


Fig. 4b

Fig. 4

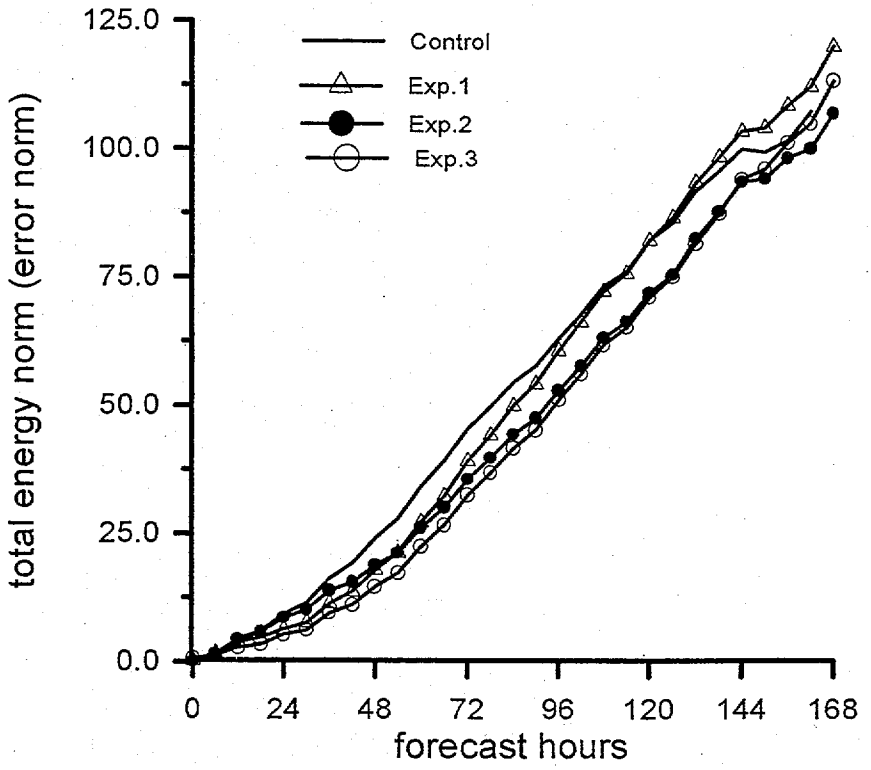

125.0
100.0
75.0
50.0
25.0
0.0
total energy norm (error norm)
Control
Exp.1
Exp.2
Exp.3
0
24
48
72
96
120
144
168
forecast hours

Fig. 5a

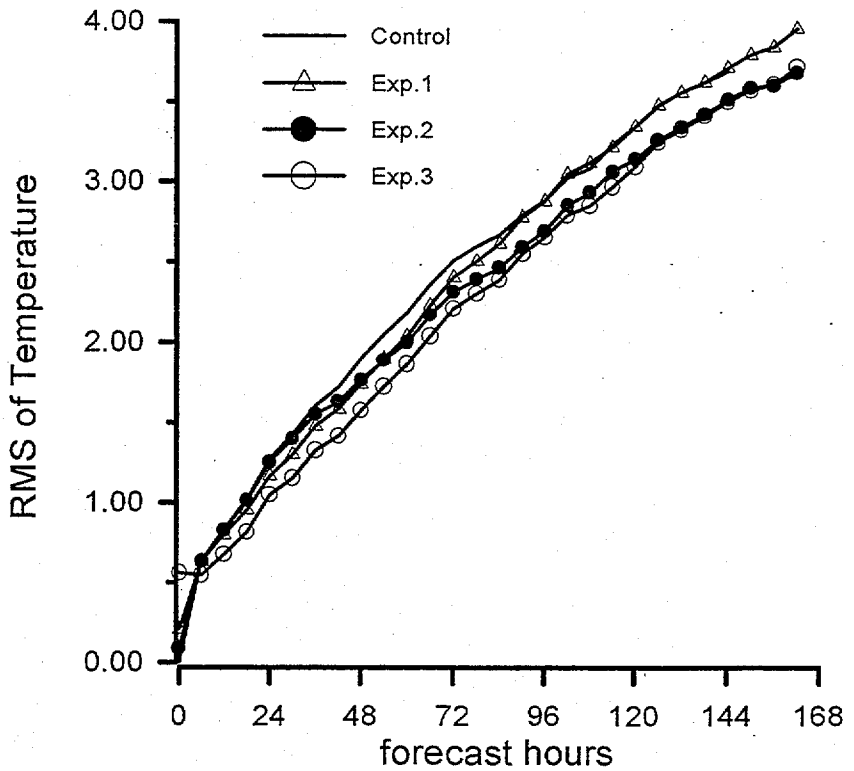

4.00
3.00
2.00
1.00
0.00
RMS of Temperature
Control
Exp.1
Exp.2
Exp.3
0
24
48
72
96
120
144
168
forecast hours

Fig. 5a

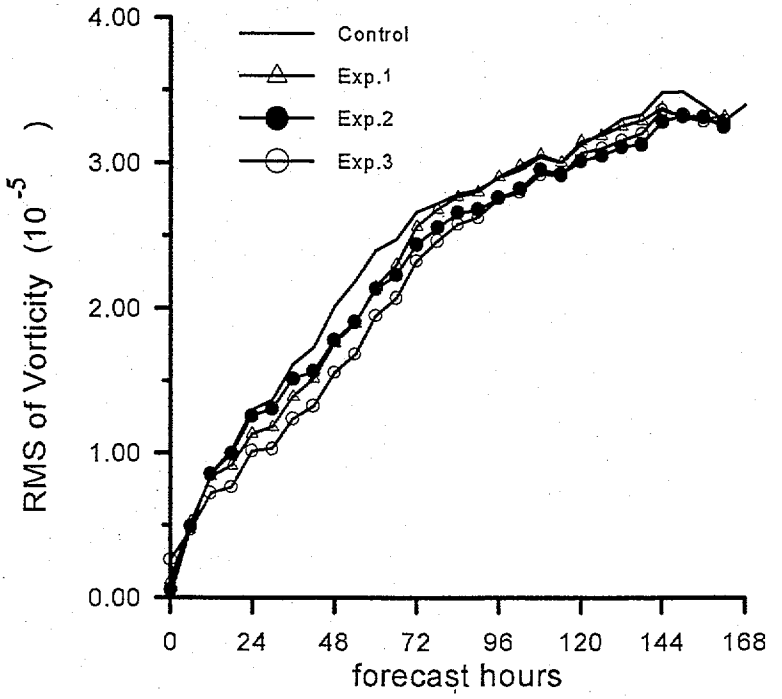

4.00
3.00
2.00
1.00
0.00
RMS of Vorticity (10^-5)
Control
Exp.1
Exp.2
Exp.3
0
24
48
72
96
120
144
168
forecast hours

Fig. 5b

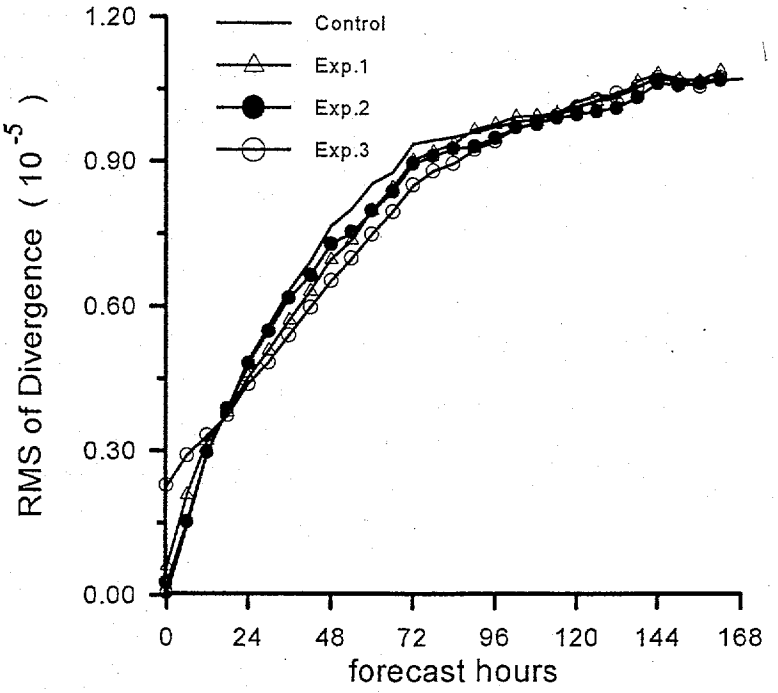
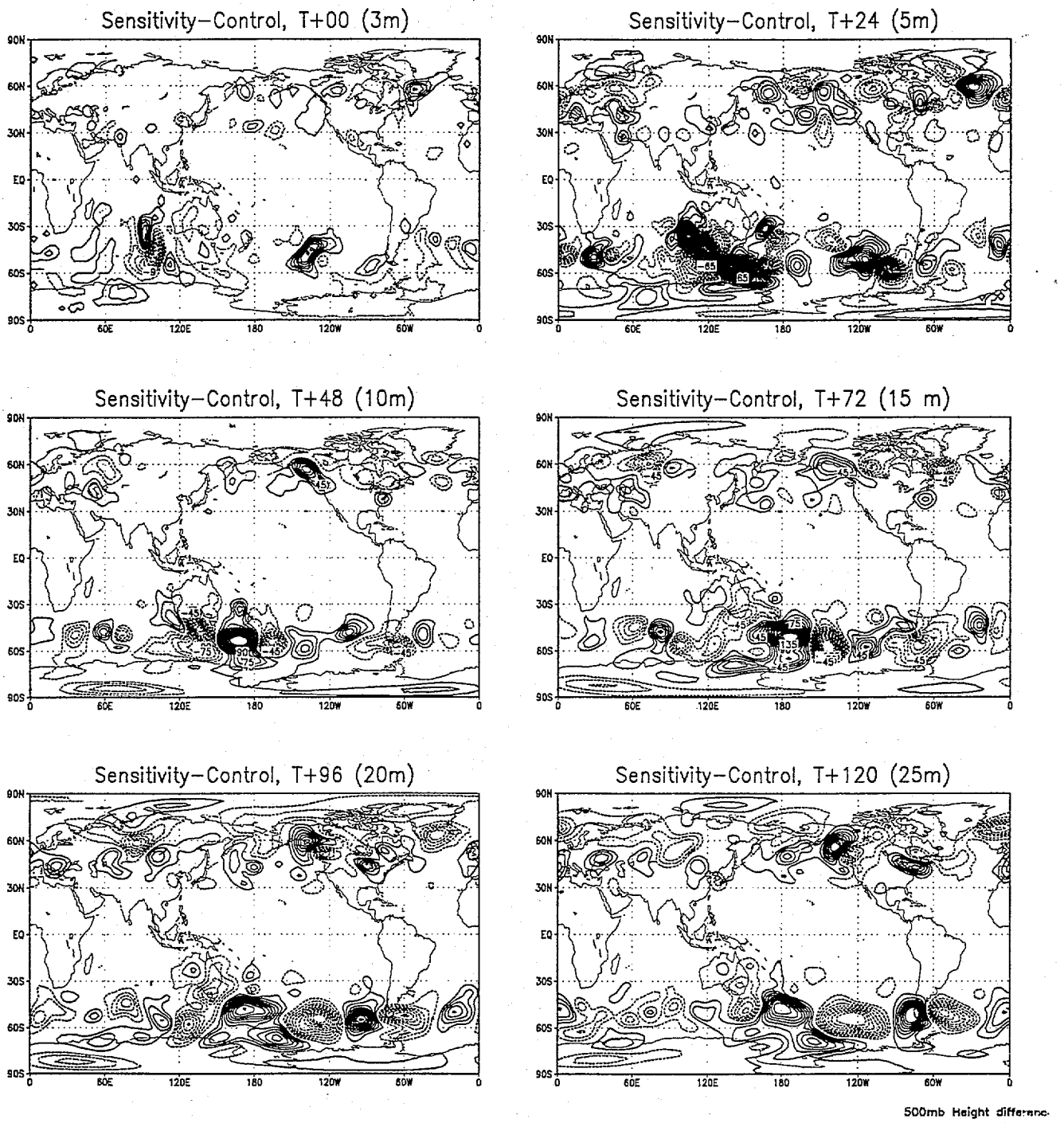
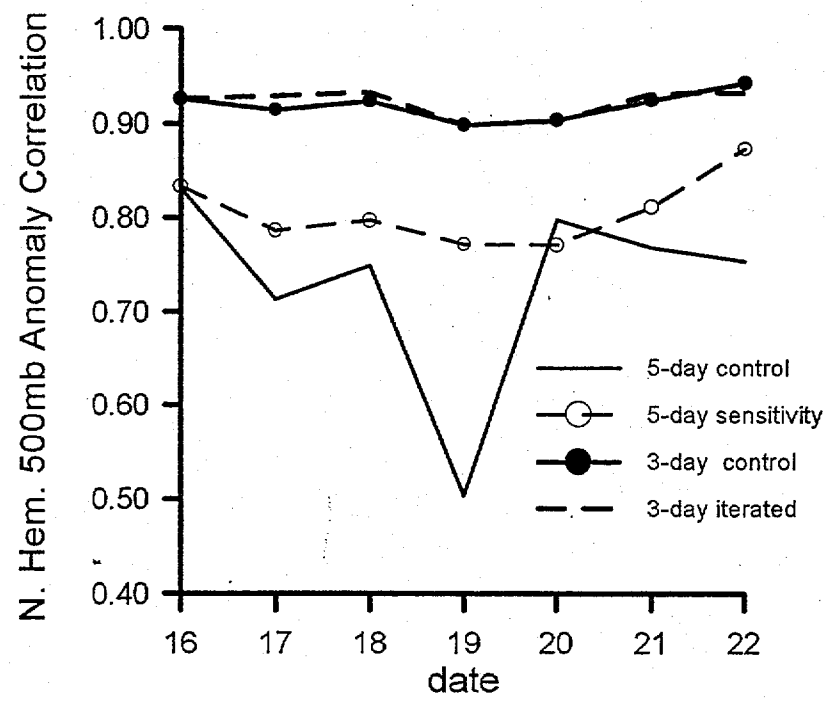

1.20
0.90
0.60
0.30
0.00
RMS of Divergence (10^-5)
Control
Exp.1
Exp.2
Exp.3
0
24
48
72
96
120
144
168
forecast hours

Fig. 5c



500mb Height difference

Fig. 6


N. Hem. 500mb Anomaly Correlation
1.00
0.90
0.80
0.70
0.60
0.50
0.40
16
17
18
19
20
21
22
date
5-day control
5-day sensitivity
3-day control
3-day iterated

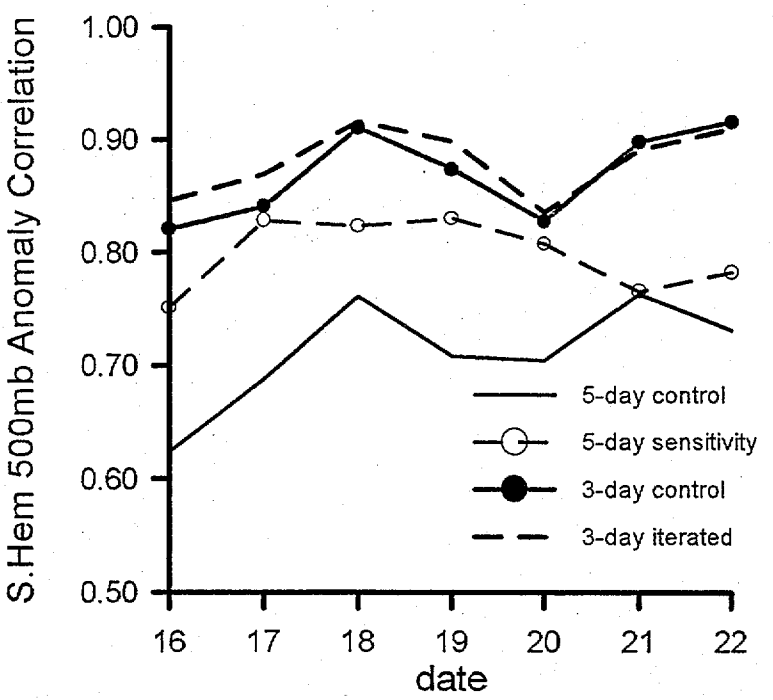
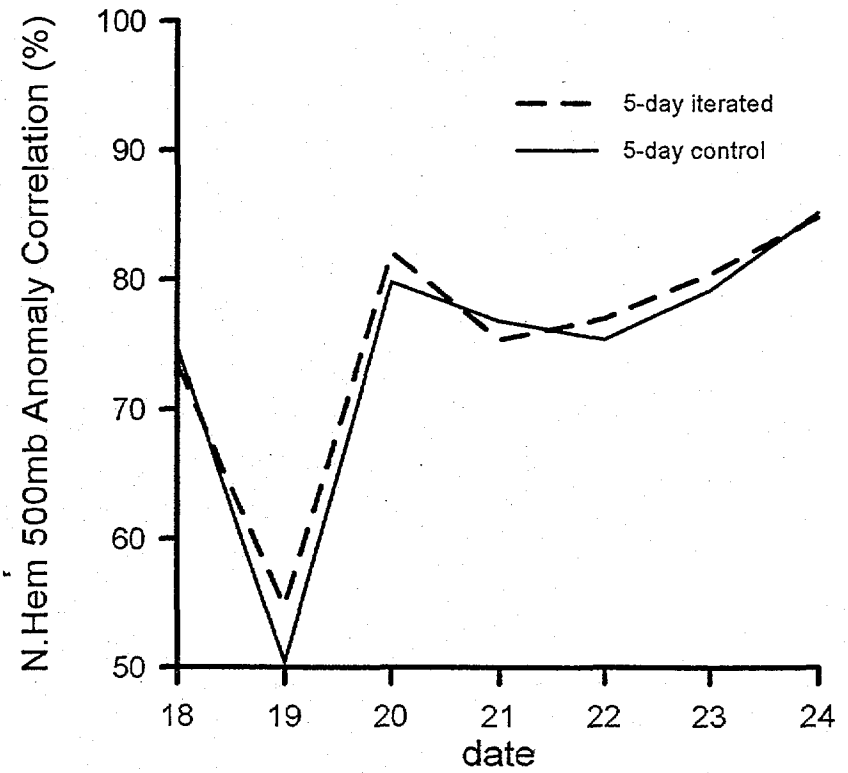

S.Hem 500mb Anomaly Correlation
1.00
0.90
0.80
0.70
0.60
0.50
16
17
18
19
20
21
22
date
5-day control
5-day sensitivity
3-day control
3-day iterated

Fig. 7


N.Hem 500mb Anomaly Correlation (%)
100
90
80
70
60
50
18
19
20
21
22
23
24
date
5-day iterated
5-day control

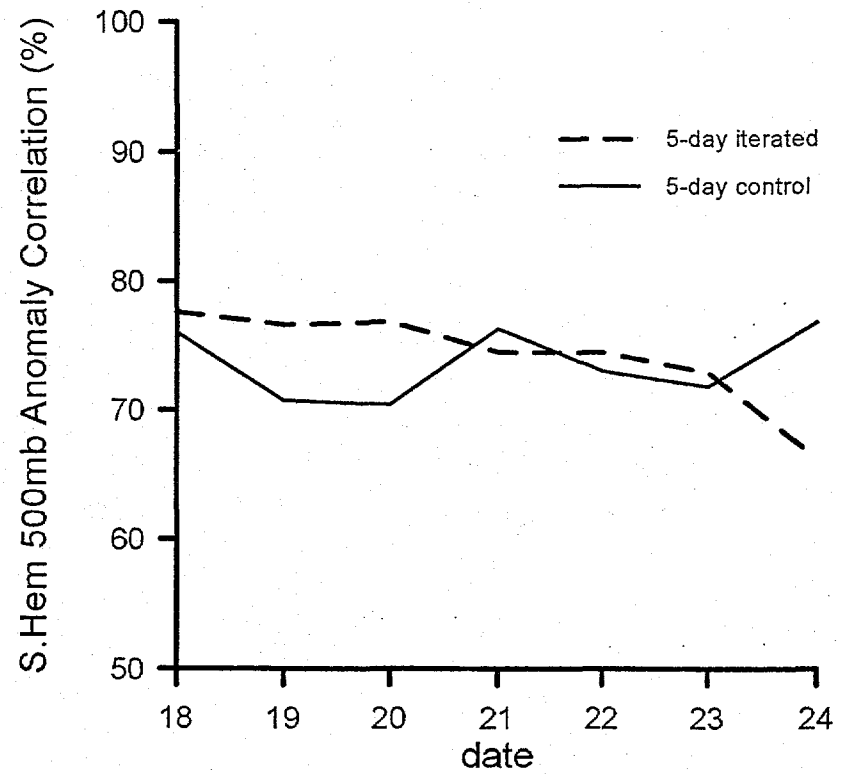
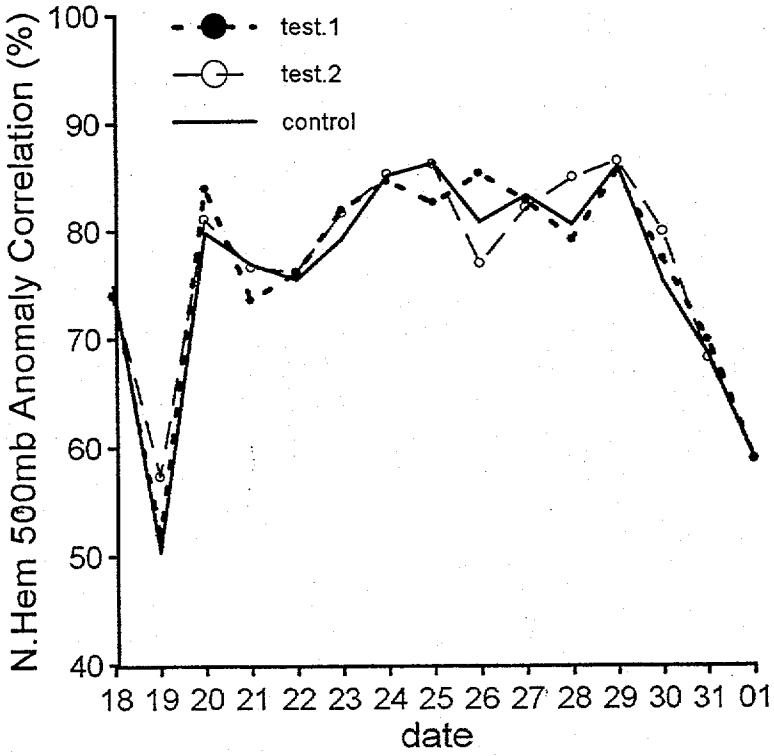

S.Hem 500mb Anomaly Correlation (%)
100
90
80
70
60
50
18
19
20
21
22
23
24
date
5-day iterated
5-day control

Fig. 8


test.1
test.2
control
N.Hem 500mb Anomaly Correlation (%)
100
90
80
70
60
50
40
18 19 20 21 22 23 24 25 26 27 28 29 30 31 01
date

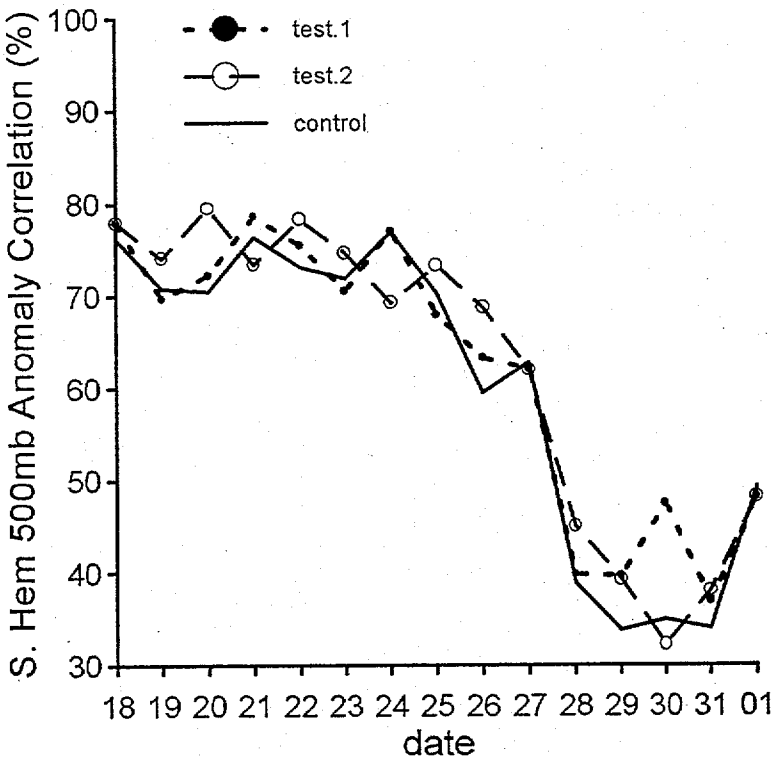
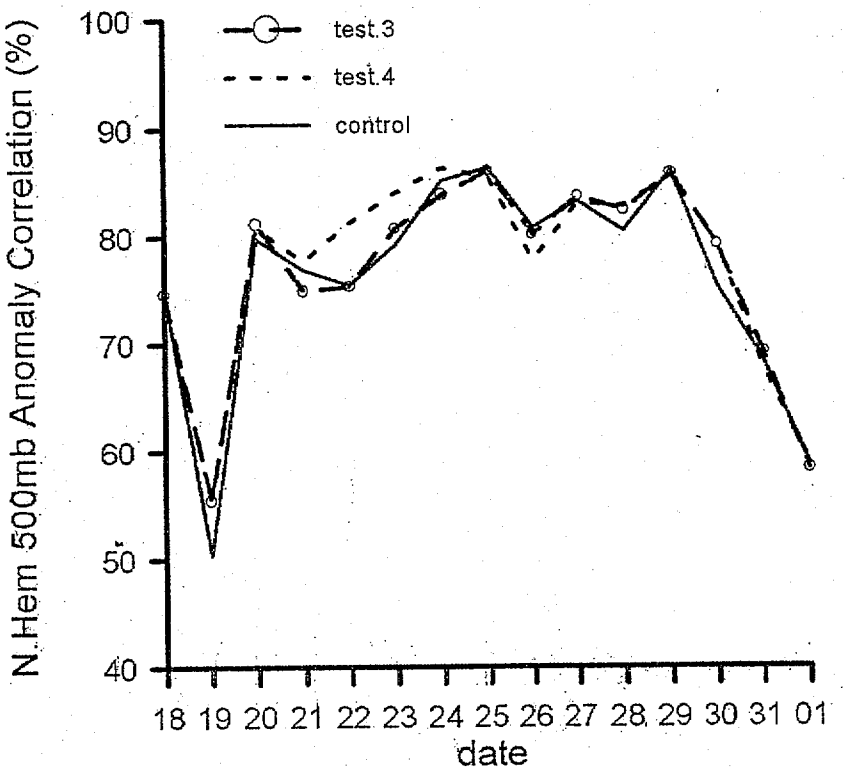

test.1
test.2
control
S. Hem 500mb Anomaly Correlation (%)
100
90
80
70
60
50
40
30
18 19 20 21 22 23 24 25 26 27 28 29 30 31 01
date

Fig. 9


N.Hem 500mb Anomaly Correlation (%)
100
90
80
70
60
50
40
test.3
test.4
control
18 19 20 21 22 23 24 25 26 27 28 29 30 31 01
date

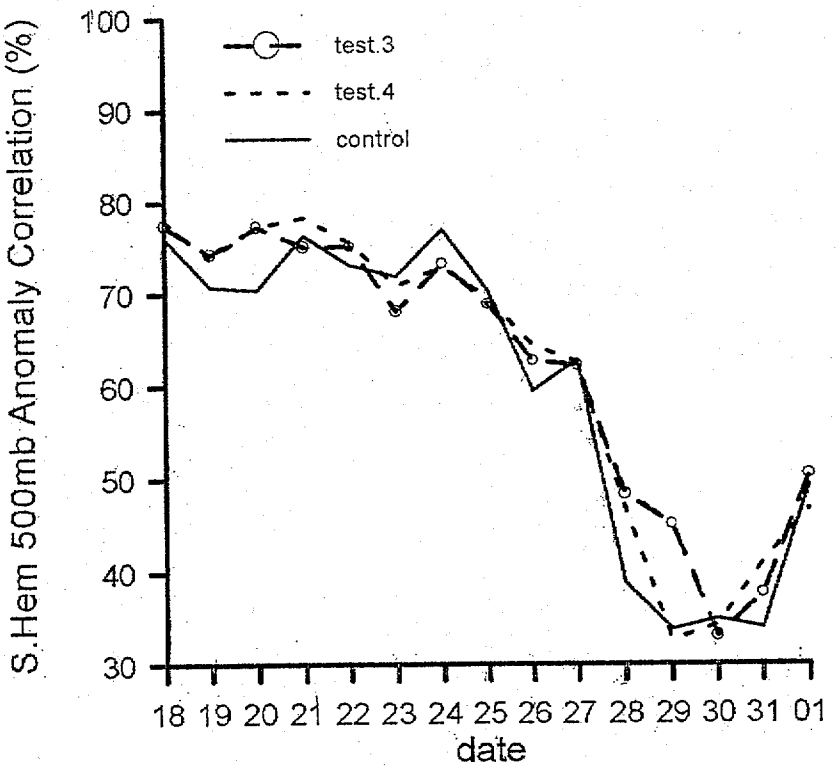
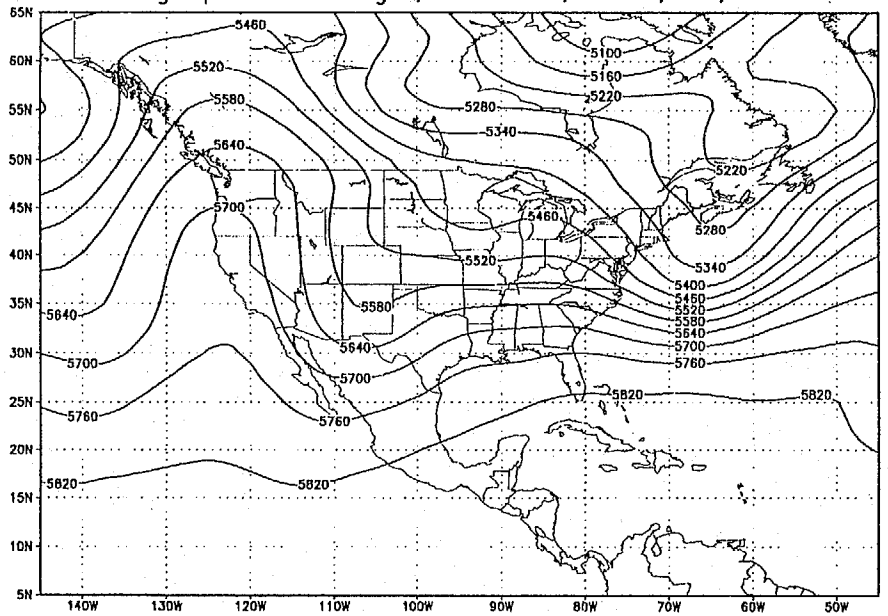

S.Hem 500mb Anomaly Correlation (%)
100
90
80
70
60
50
40
30
test.3
test.4
control
18 19 20 21 22 23 24 25 26 27 28 29 30 31 01
date

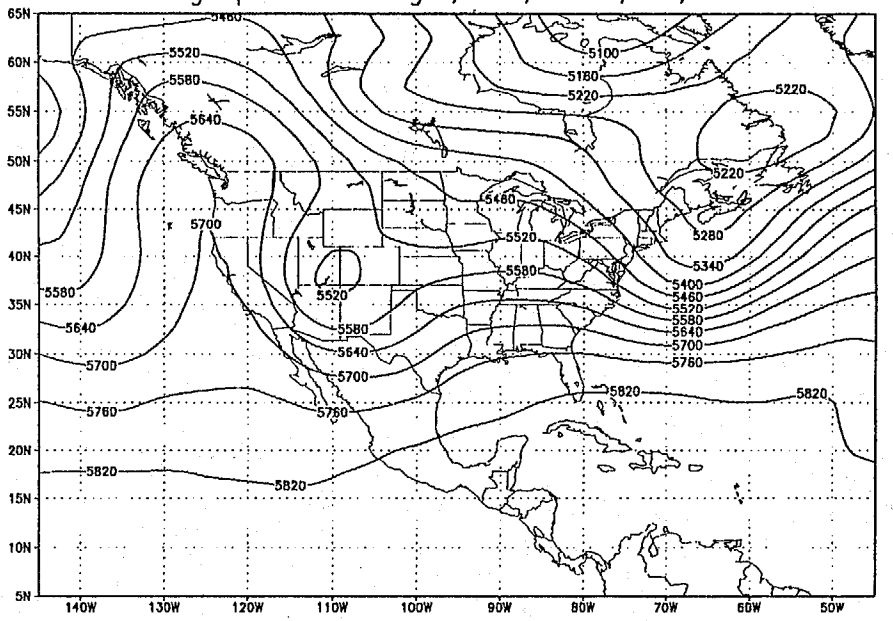
Fig. 10

500mb geopotential height,CTRL.FCST,T+120,00Z,03-24-95



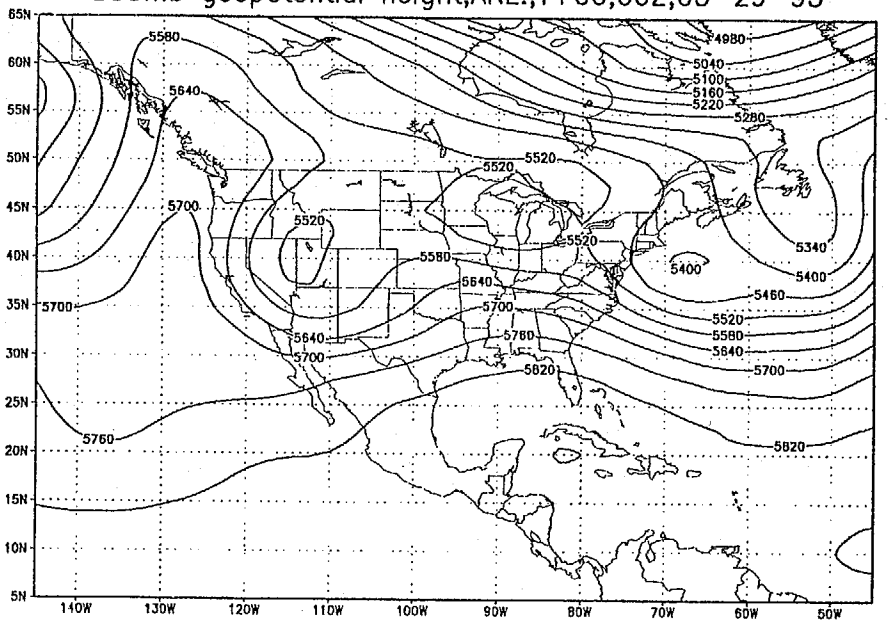
a

500mb geopotential height,FCST,T+120,00Z,03-24-95



b

500mb geopotential height,ANL.,T+00,00Z,03-29-95



c

Fig. 11

Table.1 Anomaly correlation scores of 1-5-day forecast from 0000UTC 23 February 1995 for 500mb Geopotential Height

Hemisphere	Type	day-1	day-2	day-3	day-4	day-5
Northern H.	Control	0.986	0.963	0.921	0.852	0.729
	Sensitivity	0.989	0.978	0.969	0.934	0.877
Southern H.	Control	0.933	0.820	0.762	0.670	0.592
	Sensitivity	0.943	0.833	0.767	0.725	0.666

Table.2 Comparison of the average anomaly correlation scores of 1-5-day forecast for 500mb geopotential height. Starting dates of the forecasts ranged from 18 March 1995 to 1 April 1995.

	Northern Hemisphere 1000mb					Northern Hemisphere 500 mb				
Day	Ctrl.	Test.1	Test.2	Test.3	Test.4	Ctrl.	Test.1	Test.2	Test.3	Test.4
1	0.972	0.971	0.974	0.973	0.973	0.989	0.989	0.990	0.990	0.990
2	0.937	0.934	0.942	0.938	0.938	0.965	0.963	0.966	0.966	0.966
3	0.884	0.878	0.888	0.886	0.883	0.924	0.920	0.926	0.926	0.925
4	0.788	0.787	0.797	0.796	0.793	0.852	0.849	0.859	0.858	0.859
5	0.685	0.688	0.689	0.691	0.691	0.760	0.763	0.770	0.767	0.774

	Southern Hemisphere 1000mb					Southern Hemisphere 500 mb				
Day	Ctrl.	Test.1	Test.2	Test.3	Test.4	Ctrl.	Test.1	Test.2	Test.3	Test.4
1	0.948	0.946	0.952	0.948	0.950	0.971	0.974	0.974	0.975	0.975
2	0.885	0.882	0.894	0.883	0.886	0.919	0.926	0.927	0.928	0.928
3	0.793	0.796	0.806	0.798	0.796	0.833	0.849	0.852	0.850	0.851
4	0.680	0.685	0.699	0.690	0.690	0.721	0.739	0.743	0.744	0.747
5	0.551	0.579	0.574	0.564	0.566	0.599	0.618	0.621	0.616	0.615