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OFFICE NOTE 376

"ON STATISTICS FOR VERIFICATION OF REGIONAL AND MESOSCALE MODELS"
1. CORRELATION COEFFICIENTS

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Introduction:

There are many statistics which can serve as measures of the skill of prediction. Among these are the correlation coefficient, the mean square error and the mean error. In this note we will address the characteristics of the correlation coefficient when applied to regional or mesoscale model forecasts. It will be observed that the correlation coefficient is sensitive to the spatial scale and power spectrum of the fields being verified. Additional complications arise when the correlation is computed over the small domains typical of regional or mesoscale models.

Using data sets provided by our recent work on regional data assimilation with the Nested Grid Model (NGM), some examples are given of the computation, over North America, of the correlation coefficient for analyzed and forecast 500mb fields of geopotential height, geostrophic relative vorticity and kinetic energy, and the geostrophic advection of relative vorticity. These computations show a decline in the correlation coefficient as fields with greater high-wave number variability are verified. By comparing the verification scores with those obtained for a persistence forecast, we quantify the fact that dynamical model forecasts of weather-related, differentials of the height field possess significant skill

It is recommended that verification of differential operators on the geopotential height be added to the statistics used for evaluating regional forecast. Our computations show that greater sensitivity can be achieved. Furthermore, the emphasis of smaller scales of variation can be expected to reduce dependencies of the statistics on the size of the verification domain.

2. Regional model verification.

It is common practice to compute forecast verification statistics for a regional model over specific sub-hemispheric geographic areas, e.g. eastern United States, Alaska, etc. Although the domain of verification is limited, the fields being verified may be considered to be comprised of waves of varying length, many of which have scales exceeding the largest length scale of the verification domain. On the other hand, there may be small scale components present in the field which generally will not be integer harmonics of the length of the domain. We consider below the analytic impact of these factors on certain statistics that enter into the computation of the correlation.

For a one dimensional field, using trigonometric functions to quantify the concept of spatial scale, we define an average operator,

$$\mu_F = \overline{F} = \frac{1}{D}\int_0^D (F)\,dx$$

in terms of which the correlation ρ between two fields F and G may be expressed as,

$$\rho = \frac{\overline{(F-\mu_F)(G-\mu_G)}}{\sigma_F \sigma_G}$$

with

$$\sigma_F^2 = \overline{(F-\mu_F)^2}$$

$$\sigma_G^2 = \overline{(G-\mu_G)^2}$$

The numerator of the correlation is the covariance between F and G over the verification domain $0 < x < D$. The denominator, comprised of the product of the standard deviation of F and G, serves to scale the result so that the correlation lies between -1 and +1. The correlation is indeterminate if the variance of either F or G vanishes.

2a. The Mean:

Suppose that F is a forecast field that can be represented by a single waveform,

$$F = A_F \cos\left(\frac{2\pi}{L}(x - \mathcal{X})\right)$$

One finds that the mean, $\mu_F = \bar{F}$ is given by,

$$\bar{F} = A_F\left(\frac{\sin b}{b}\cos a + \frac{(1 - \cos b)}{b}\sin a\right)$$

with

$$b = \frac{2\pi D}{L} \qquad\qquad a = \frac{2\pi \mathcal{X}}{L}$$

Thus, the computed mean value of F <u>in the verification region</u> (0,D) will depend on the relative size of the domain and the scale, or wavelength, of the field, as well as upon the phase of the waveform with respect to the boundary of the domain. For small values of D/L, the "mean value" of F may lie close to +/- A_F. When D is close (but not equal) to L, the mean value of F will be calculated to be about +/- 0.1 A_F. For D near 2 L, the computed value of the mean will be less than +/- .05 A_F.

2b The Variance:

The variance of the waveform F is given by,

$$Var(F) = \overline{F^2} - \mu_F^2$$

Computation yields,

$$\overline{F^2} = \frac{A_F^2}{4b}\{2b + \cos 2a \sin 2b + \sin 2a(1 - \cos 2b)\}$$

where again

$$b = \frac{2\pi D}{L} \qquad\qquad a = \frac{2\pi \mathfrak{x}}{L}$$

We note that, if D = n L with n an integer, then the mean vanishes,and $Var(F)=A_F^2/2$; otherwise, the phase of the waveform with respect to the verification domain influences the values computed for both the mean and variance.

2c The Covariance:

Consider two fields F and G defined by the waveforms,

$$F = A_F \cos\left(\frac{2\pi}{L_F}(x - \mathfrak{x}_F)\right)$$

$$G = A_G \cos\left(\frac{2\pi}{L_G}(x - \mathfrak{x}_G)\right)$$

The covariance of F and G may calculated with the aid of,

$$\begin{aligned}\overline{FG} = A_F A_G \Big\{ &\cos a_F \cos a_G \left(\frac{\sin(b_F - b_G)}{2(b_F - b_G)} + \frac{\sin(b_F + b_G)}{2(b_F + b_G)}\right) \\ &+ \sin a_F \sin a_G \left(\frac{\sin(b_F - b_G)}{2(b_F - b_G)} - \frac{\sin(b_F + b_G)}{2(b_F + b_G)}\right) \\ &+ \cos a_F \sin a_G \left(\frac{(1 - \cos(b_F + b_G))}{2(b_F + b_G)} - \frac{(1 - \cos(b_F - b_G))}{2(b_F - b_G)}\right) \\ &+ \sin a_F \cos a_G \left(\frac{(1 - \cos(b_F - b_G))}{2(b_F - b_G)} + \frac{(1 - \cos(b_F + b_G))}{2(b_F + b_G)}\right)\Big\}\end{aligned}$$

in which,

$$a_F = \frac{2\pi \ell_F}{L_F} \qquad a_G = \frac{2\pi \ell_G}{L_G}$$

$$b_F = \frac{2\pi D}{L_F} \qquad b_G = \frac{2\pi D}{L_G}$$

One may write the covariance as,

$$Cov(F,G) = \overline{FG} - \mu_F \mu_G$$

If F and G have the same wavelength, i.e. $b_F = b_G = b$, then

$$\overline{FG} = \frac{A_F A_G}{4b}\{2b\cos(a_F - a_G) + \cos(a_F + a_G)\sin 2b + \sin(a_F + a_G)(1 - \cos 2b)\}$$

If further D = n L, then b = $2\pi n$ and

$$\overline{FG} = \frac{A_F A_G}{2}\cos(a_F - a_G)$$

which indicates a simple relationship between covariance (or correlation) and the relative phase of the correlated fields. In the more general case, no such simple relationship exists.

If F and G have different wavelengths, each of which is a harmonic of the range D of integration, then $\overline{FG} = 0$. Note however that even if F and G are orthogonal functions over some fundamental interval, they will generally have a non-vanishing covariance over an arbitrary sub-domain D.

3. Correlation of superimposed waveforms:

We have seen in section 2. that, when the computation is done over an arbitrary domain, the correlation statistic is a complex function of phase angle and of the scale of the field being verified and the size of the verification domain. In this section, we draw attention to the dependence of the correlation of complex waveforms on the power spectrum of the spectral components comprising the waveforms.

We restrict the argument to the case in which the computation is performed over a domain that is a "fundamental" for all the elements in the waveform. Let F and G be the two fields to be correlated, with

$$F = \sum_{n=1}^{N} F_n \cos\left(2\pi n\left(x - \mathcal{X}_{nF}\right)/L\right)$$
$$G = \sum_{n=1}^{N} G_n \cos\left(2\pi n\left(x - \mathcal{X}_{nG}\right)/L\right)$$

and let the average operator be defined by,

$$\overline{F} = \frac{1}{L}\int_0^L F\,dx$$

For the fields F and G, one obtains:

$$\overline{F} = \overline{G} = 0$$

$$\sigma_F^2 = var(F) = \overline{F^2} = \frac{1}{2}\sum_{n=1}^{N} F_n^2$$

$$\sigma_G^2 = var(G) = \overline{G^2} = \frac{1}{2}\sum_{n=1}^{N} G_n^2$$

$$Cov(FG) = \overline{FG} = \frac{1}{2}\sum_{n=1}^{N} F_n G_n \cos\delta_n$$

$$\delta_n = +/-\left(\mathcal{X}_{nF} - \mathcal{X}_{nG}\right)$$

$$Cor(FG) = Cov(FG)/\sigma_F\sigma_G$$
$$= \sum_{n=1}^{N} F_n G_n \cos\delta_n / P$$

in which

$$P \equiv \left\{\left(\sum_{n=1}^{N} F_n^2\right)\left(\sum_{n=1}^{N} G_n^2\right)\right\}^{\frac{1}{2}}$$

If we define

$$\rho_i = \cos\delta_i \qquad\qquad w_i = F_i G_i / P$$

then

$$Cor(FG) = \sum_{n=1}^{N} w_n \rho_n$$

The factor w_i is proportional to ratio of the power spectral density in specific wave numbers to the total power. We may call it the correlation weight. For many meteorological fields the bulk of the power is found in the planetary waves; the power spectrum falls off rapidly in the higher wavenumbers. The correlation weights w_n will reflect this property of the power spectrum, and consequently the correlation coefficient will be dominated by the characteristics of the planetary scale components of the field.

For global models this property of the correlation coefficient may be accounted for by using spectral decomposition of the fields and evaluating the correlation for specific portions of the spectrum (Murphy and Epstein,1989.) For regional models verified over sub-hemispheric domains approximate methods for achieving such spectral decomposition have been suggested by Bettge and Baumhefner (1980) and Errico (1985.)

Although basic meteorological fields are characterized by the dominance of the planetary scale features of the circulation, there are other fields (e.g. relative vorticity, static stability, specific humidity and precipitation) that possess relatively greater variability in smaller scales; indeed this is a significant reason for our interest in the development of small-synoptic and mesoscale prediction models. By focusing evaluation on these differentiated fields a more meaningful verification of small synoptic and mesoscale forecasts may be achieved. One must not ignore however the greater uncertainty associated with the analysis of these quantities.

4. An Example:

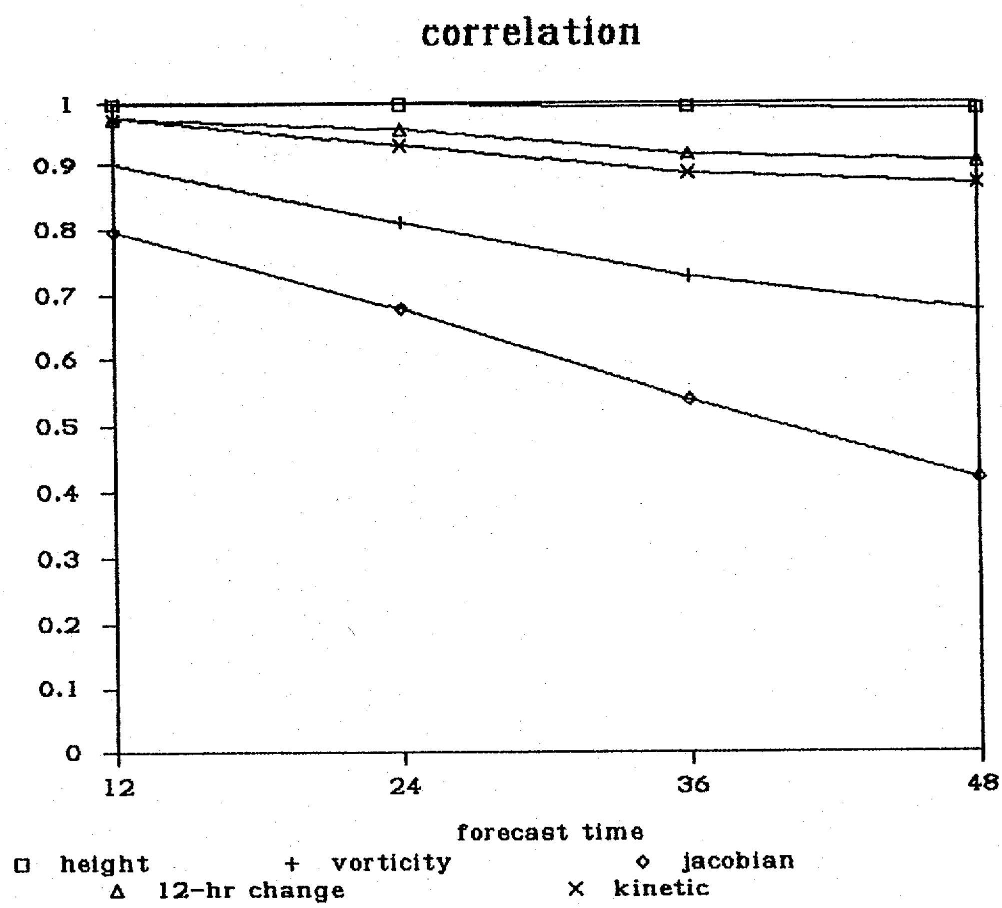
Our recent experimental runs of the Regional Data Assimilation System based on the revised Nested Grid forecast model have provided a readily accessible data archive that we have used to exhibit some properties of the correlation coefficient as a measure of forecast skill.

For the quoted statistics (a complete set is attached), we used the 500 mb height field analyses and forecasts for the 48 hour period beginning 00Z 17 March 1990. In order to assess the anticipated variation in forecast skill with spatial scale, we have calculated finite difference approximations of the geostrophic kinetic energy and relative vorticity, and of the Jacobian operator giving the geostrophic advection of geostrophic relative vorticity. (In computing these terms we neglected precise scaling by the coriolis and map-scale factors.) The statistical evaluation was done over the North American region shown in the subsequent maps.

Let's first examine the correlation coefficient between forecasts and corresponding verifying analyses, computed at 12-hour intervals:

	Height	Kin En	Vorticity	Jacobian
12-hour	.998	.974	.900	.796
24-hour	.997	.931	.810	.677
36-hour	.994	.887	.727	.538
48-hour	.990	.871	.674	.419

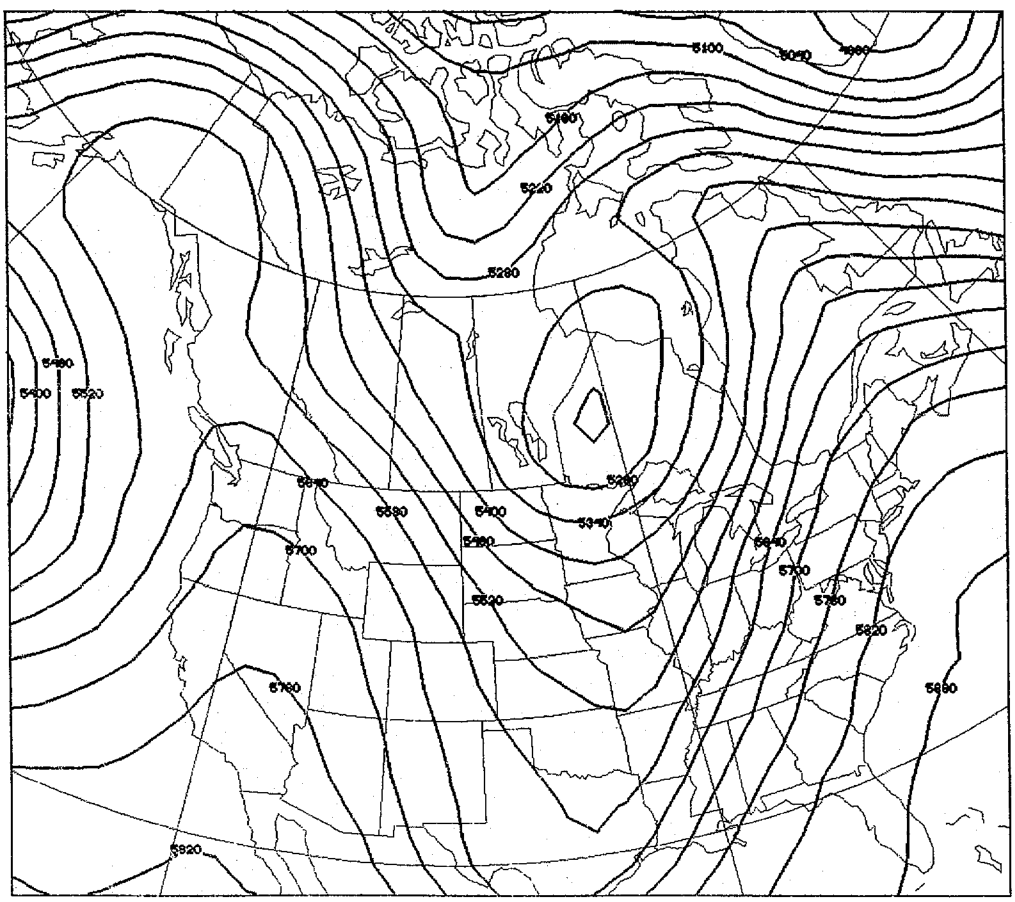
These correlation coefficients (see graph below) show a decline of forecast skill that might have been anticipated. For an April case, the power spectum of 500 mb geopotential height was found (Gerrity and Parrish, 1982) to decline about two orders of magnitude as the global wave number changed from 6 to 20 -- wavelength change from 6000 km to 2000 km. Thus, the skill of the forecast in predicting waves with scale about 6000 km is weighted about 100 times more heavily than waves with scale about 2000 km in evaluating the correlation coefficient for the height. This weight differential shifts to more nearly one-to-one in the vorticity, because of the wave-number squared multiplier in the vorticity. For the Jacobian - i.e. advection of vorticity, the weight assigned to the 2000 km wave most likely exceeds that assigned to the 6000 km wave.



So, although the forecast height is very highly correlated to the analyzed height, this can be attributed to the dominance of the energy content of the well-predicted, large scale components of the field. For weather forecasting, it is important to predict differential aspects of the height field. For these components, we see that the forecast skill declines appreciably with decreasing scale and with the length of the forecast. To account for this, it seems important to eschew sole use of the height correlation in forecast evaluation. Greater focus on the correlation of differential forms may provide better measures of the value of a forecast for weather prediction.

In the figure below, we show the 500 mb height field analyzed over the North American verification domain for 00Z 17 Mar 90, the initial time for the forecasts verified. Note that the synoptic state may be characterized as being a low-index flow regime; one for which, persistence might be a good predictor. The correlation coefficient for the persistence forecast of the height was calculated with the results: .967, .907, .850 and .821 for 12, 24, 36 and 48 hour forecast lengths. At the other extreme, the persistence forecast of the Jacobian was almost uncorrelated with the verifying analyses. We computed the correlation coefficients: .059, .101, .018 and .078 for 12, 24, 36 and 48 hours. Thus, even though the correlation coefficients calculated for the RAFX forecast of the jacobian (advection of relative vorticity) were low, they provided a significant improvement over a persistence forecast.

INITIAL 500 MB HEIGHT 00Z 17 MARCH 1990



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Height

HOUR	mean obs	mean fcst	standard deviation obs	standard deviation fcst	cor	std dev error
12	5498.3	5506.4	235.1	230.8	0.998	13.6
24	5482.9	5494.8	236.4	232.4	0.997	18.8
36	5472.9	5480.3	241.7	238.7	0.994	27.1
48	5478.9	5474.3	250.5	248.4	0.99	34.8

Vorticity

HOUR	mean obs	mean fcst	standard deviation obs	standard deviation fcst	cor	std dev error
12	-0.3	-0.6	18.5	16.6	0.9	8.1
24	-0.3	-0.5	17.4	15.1	0.81	10.2
36	0.2	0	17.1	14.8	0.727	12
48	0.8	1	17.1	14.7	0.674	13.1

Jacobian

HOUR	mean obs	mean fcst	standard deviation obs	standard deviation fcst	cor	std dev error
12	-0.3	8.9	1673.9	1328	0.796	1012.7
24	18.3	16.3	1450.4	1217.6	0.677	1093.6
36	27.2	36.1	1517.1	1068.6	0.538	1303.5
48	22	40.1	1543.6	1029	0.419	1452.2

Height Change over 12 hours

HOUR	mean obs	mean fcst	standard deviation obs	standard deviation fcst	cor	std dev error
12	-26.7	-18.7	59.5	58.3	0.973	13.6
24	-15.4	-11.6	59.9	57.6	0.957	17.3
36	-10.1	-14.5	60.7	56.2	0.917	24.3
48	6	-5.9	63.3	56.9	0.904	27.1

Kinetic

HOUR	mean obs	mean fcst	standard deviation obs	standard deviation fcst	cor	std dev error
12	42	40.7	40.9	38.5	0.974	9.4
24	41.6	40.1	39	36.4	0.931	14.2
36	41.8	40	41.1	37.4	0.887	19
48	42.4	41.3	46.6	42.4	0.871	23

Persistence forecast Height

HOUR	mean obs	mean fcst	standard deviation obs	fcst	cor	std dev error
12	5498.3	5525.1	235.1	227.7	0.967	59.5
24	5482.9	5525.1	236.4	227.7	0.907	100.2
36	5472.9	5525.1	241.7	227.7	0.85	129.3
48	5478.9	5525.1	250.5	227.7	0.821	144.7

Persistence forecast Vorticity

HOUR	mean obs	mean fcst	standard deviation obs	fcst	cor	std dev error
12	−0.3	−1.1	18.5	18.4	0.385	20.5
24	−0.3	−1.1	17.4	18.4	0.165	23.1
36	0.2	−1.1	17.1	18.4	−0.015	25.3
48	0.8	−1.1	17.1	18.4	0.129	23.5

Persistence forecast Jacobian

HOUR	mean obs	mean fcst	standard deviation obs	fcst	cor	std dev error
12	−0.3	28.5	1673.9	1582.2	0.059	2234.3
24	18.3	28.5	1450.4	1582.2	0.101	2035.1
36	27.2	28.5	1517.1	1582.2	0.018	2172.1
48	22	28.5	1543.6	1582.2	0.078	2122.3