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A SPLIT QUASI-LAGRANGIAN METHOD FOR USE IN A REGIONAL BAROTROPIC MODEL

TENG JIAMO
DEVELOPMENT DIVISION

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Teng Jiamo*
National Meteorological Center
National Weather Service. NOAA

ABSTRACT

An economical advection scheme with linear high order accuracy is described. A splitting technique is used to solve the equations of a barotropic model. The horizontal advection process is integrated by a quasi-Lagrangian technique which uses cubic splines and deviates from past work by not employing the upstream method. The adjustment process is integrated with a forward-backward scheme.

The application of this numerical method to a regional barotropic model shows that it can take full advantage of the splitting method by allowing a long time step for advection while maintaining high accuracy through the use of a cubic spline interpolation line (row) by line (row).

Two approaches to the treatment of lateral boundary conditions and alternate schemes for the calculation of the adjustment process are discussed.

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* Visiting scientist from the Sichuan Province Weather Bureau, The People's Republic of China.

1. INTRODUCTION

From the 1970's the quasi-Lagrangian method has come into use again (Purnell 1976; Mahrer 1978; Mathur 1979,1983; Robert 1981, 1982; Bates and McDonald 1982; Bates 1984). Investigations of the properties of various quasi-Lagrangian advection schemes have been given by Crowley (1968), Purnell (1976), Long and Pepper (1981). The upstream numerical scheme is most often used in quasi-Lagrangian models, because it makes possible interpolation from grid points to an intermediate point and permits the use of longer time steps than many other schemes.

In principle, one may improve the accuracy of the quasi-Lagrangian computation through the use of a convergent sequence of iterations. The length of the time step and the number of iterations must be balanced with the computational cost and attained accuracy. We attempted some experiments in this manner, but found that not all iteration sequences are, in fact, convergent. This experience led us to the use of a splitting method and the numerical technique reported in this paper.

When a splitting scheme is chosen, a calculation of the effects of different physical factors, one at a time, usually

leads to an increase in the truncation error. But if the time step for the advection process is not too long, the truncation error is not significant. In fact, for a regional model using a splitting, quasi-Lagrangian method, the time step should not be too long because the properties of a particle could not be computed. The reason is both numerical and physical. If the time step for the advection process is excessive, first, for a regional model the position of a particle near the inflow boundary one time step ago could be too far outside the region; second, in the quasi-Lagrangian advection scheme it is possible that two or more particles which have different initial values arrive at the position at the same time, but there can only be one value and the point of origin can be completely miscalculated, as, for example, in the case of very strong distortion field.

Section 2 of this paper discussed the basic form of an algorithm for a quasi-Lagrangian method that uses a cubic spline interpolation with a downstream scheme. Section 3 and 4 present adjustment process schemes and methods for handling lateral boundary conditions, that have been considered. In section 5, a 48 hour forecast from a barotropic model is compared with the analysis fields.

2. THE ADVECTION SCHEME

Our choice of the splitting technique is partially motivated by the desire to avoid iteration for high interpolation accuracy. The solution to the governing equations (1) is divided into two main stages: the adjustment stage (2) and the advection stages (3)

$$\begin{cases} \dfrac{du}{dt} = -m\dfrac{\partial\phi}{\partial x} + fv \\ \dfrac{dv}{dt} = -m\dfrac{\partial\phi}{\partial y} - fu \\ \dfrac{d\phi}{dt} = -m^2\phi\left[\dfrac{\partial}{\partial x}\left(\dfrac{u}{m}\right) + \dfrac{\partial}{\partial y}\left(\dfrac{v}{m}\right)\right] \end{cases} \tag{1}$$

$$\begin{cases} \dfrac{\partial u}{\partial t} = -m\dfrac{\partial\phi}{\partial x} + fv \\ \dfrac{\partial v}{\partial t} = -m\dfrac{\partial\phi}{\partial y} - fu \\ \dfrac{\partial\phi}{\partial t} = -m^2\phi\left[\dfrac{\partial}{\partial x}\left(\dfrac{u}{m}\right) - \dfrac{\partial}{\partial y}\left(\dfrac{v}{m}\right)\right] \end{cases} \tag{2}$$

$$\begin{cases} \dfrac{du}{dt} = 0 \\ \dfrac{dv}{dt} = 0 \\ \dfrac{d\phi}{dt} = 0 \end{cases} \tag{3}$$

This section will be devoted to the solution of (3); the adjustment scheme will be discussed in section 3.

We use a downstream quasi-Lagrangian method with cubic spline interpolation as an advection scheme for the advection stage (3). Integrating over the trajectory of a particle which comes from a grid point (i,j) at time $n\Delta t$ and arrives at a point P at time $(n+1)\Delta t$, we have

$$A(i\Delta x + u\Delta t,\ j\Delta y + v\Delta t,\ (n+1)\Delta t) = A(i\Delta x,\ j\Delta y,\ n\Delta t) . \qquad (4)$$

Where A is a property of the fluid which satisfies (3), and the value $A(i\Delta x,\ j\Delta y,\ (n+1)\Delta t)$ can be estimated from interpolation.

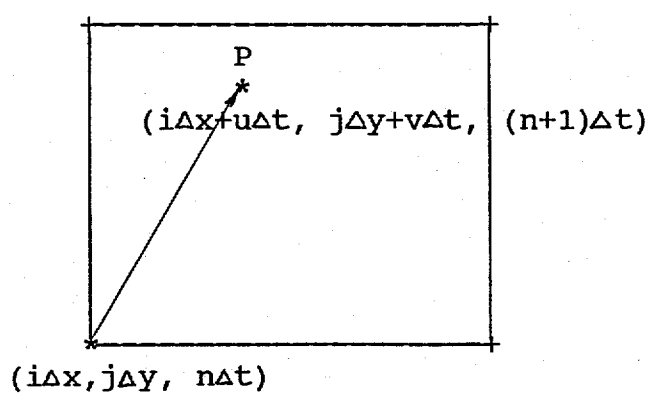

Fig. 1

For a regional model, as the reason mentioned in section 1, the following approximate limitation of time step should be introduced

$$U_{max} \Delta t / \Delta x \leqslant b \tag{5}$$

Where Umax is maximum wind speed, Δt is time step; Δx is grid interval; b is a constant which depends upon advection difference scheme and a requirement of boundary scheme. For an Eulerian difference scheme the value of b should be 1.0 and for a quasi-Lagrangian method it can be greater than 1.0. The requirement of boundary means that, if there is no buffer zone between boundary and internal area, the value of b should be 1.0.

The one dimensional cubic spline interpolation formulas are

$$\begin{aligned} f(x_i) = {} & f(x_j)\frac{1}{h_j^3}(x_{j+1} - x_i)^2 \, [\, 2(x_i - x_j) + h_j \,] \\ & + f(x_{j+1})\frac{1}{h_j^3}(x_i - x_j)^2 \, [2(x_{j+1} - x_i) + h_j] \\ & + m(x_j)\frac{1}{h_j^2}(x_{j+1} - x_i)^2 \, (x_i - x_j) \\ & - m(x_{j+1})\frac{1}{h_j^2}(x_i - x_j)^2 \, (x_{j+1} - x_i) \end{aligned} \tag{6}$$

$$
\begin{cases}
m(x_1) = 0 \\
\dfrac{1}{h_{j-1}} m(x_{j-1}) + 2\left(\dfrac{1}{h_{j-1}} + \dfrac{1}{h_j}\right) m(x_j) + \dfrac{1}{h_j} m(x_{j+1}) \\
\qquad = 3\left[\dfrac{f(x_j) - f(x_{j-1})}{h_{j-1}} + \dfrac{f(x_{j+1}) - f(x_j)}{h_j}\right] \\
m(x_n) = 0
\end{cases}
\tag{7}
$$

Where x_i is an interpolated point, $x_i \in [x_j, x_{j+1}]$; $h_j = x_{j+1} - x_j$; $f(x_i)$ is a interpolated value at the grid point x_i ; j and i are subscript of data points (the following parcel points) and interpolated points (the following grid points) respectively. Since h_j ($j = 1,..n-1$) can be different from each other, it enables interpolation from irregular points to regular points. (7) is a tridiagonal system and requires the function at all data points for any interpolated point. If a cubic spline interpolation with downstream quasi-Lagrangian method is used and if there is a relationship between locations of parcel points and grid points, (6) can be run for a whole line. It will then be possible to save computer time greatly.

A relationship between the parcel points and the grid points (interpolated points) for downstream quasi-Lagrangian

advection scheme is very clear in the case of b=1. If $u(i\Delta x, n\Delta t) > 0$, the ith grid point is located between (j-1)th parcel point and jth parcel point which were located at (i-1)th grid point and ith grid point at the time $n\Delta t$ respectively. Otherwise, (i.e. $u<0$) the ith grid point should located between the jth parcel point and (j+1)th parcel point which were located at the ith grid point and (i+1)th grid point at the time $n\Delta t$ respectively (Fig. 2).

```
                j-1                     j     j+1
                                 -------->
(a)       ---*-+----------+-------*--+--*----> x
                i-1         i         i+1

                j-1  j                        j+1
                     <--------
(b)       ---*-+--*-------+----------+----*--> x
                i-1         i         i+1
```

Fig. 2 (a) $u_i > 0$, (b) $u_i < 0$

* notes position of ith particle at $(n+1)\Delta t$
\+ notes position of ith particle at $n\Delta t$

Since in (6) $X_i \in [X_j, X_{j+1}]$, we have the relationship

$$j = i-1 \qquad \text{when } u_i > 0$$
$$j = i \qquad\qquad u_i < 0 \quad . \qquad (8.a)$$

In the case of b = 2 (Fig. 3), it is not difficult to find the relationship. They are

```
                 j-2          j-1   j     j+1     j+2
                        --------->
(a)     --+----*--+-------+-*----*+-----*-+-----*-
         i-2     i-1      i      i+1     i+2

         j-1   j    j+1          j+2
                     <-----------
(b)     --+*----*-+---*---+-------+*------+-------
         i-2     i-1      i      i+1     i+2
```

Fig.3 (a) $u_i > 0$ and $u_{i-1} > Umax/b$
(b) $u_i < 0$ and $u_{i-1} < -Umax/b$
* and + as Fig.2

$$
\begin{array}{llll}
u_i > 0: & & & \\
 & j = i-1 & \text{when} & u_{i-1} < Umax/b \\
 & j = i-2 & & u_{i-1} > Umax/b \\
u_i < 0: & j = i & \text{when} & |u_{i+1}| < Umax/b \\
 & j = i+1 & & |u_{i+1}| > Umax/b
\end{array}
\qquad (8.b)
$$

As stated above, j is the subscript of parcel points at the time $(n+1)\Delta t$, to which the particles have come from the corresponding grid point. In order to maintain the monotone order of parcel points, the above limitation of time step (5) should be changed into

$$U_{max}\Delta t/\Delta x \leqslant (b - e) , \qquad (9)$$

where e is a constant (0.0 - b/2) that accounts for the strongest horizontal wind shear. In synoptic scale or meso-α scale systems there is not a very strong shear in horizontal wind field. So the value of e usually is very small.

Obviously, two dimensional cubic spline interpolation is more complex than one dimensional. After employing the splitting technique in different physical processes of a barotropic model, the advection scheme with upstream quasi-Lagrangian method, which uses two dimensional cubic spline interpolation, has the linear 4th order accuracy (Crowley 1968). But, as mentioned before, this scheme needs much more computer time, thus, an advection scheme which requires less computer time and does not lose too much accuracy is needed.

SCHEME A: this scheme consists of two main steps: (1) after choosing two parcel points, which are close to the ith column and located on each side of ith column, two point interpolation is used to calculate the position of the interpolated point at the ith column and value at this point; (2) one dimensional cubic spline interpolation is used in y-direction.

SCHEME B: (splitting scheme) The advection scheme

is constructed following Marchuk's method which means the advection in x-direction is followed by that in y-direction. It is very clear that this scheme with cublic spline interpolation can save much more computer time than the upstream quasi-Lagrangian method with two dimensional cubic spline interpolation. But, it would introduce a new truncation error which depends closely on the advection time step.

SCHEME C: (upstream scheme) The upstream quasi-Lagrangian method with two dimensional cubic spline interpolation is used as an advection scheme.

Fig. 4, 5 and 6 show 12-48 hour forecasts of a barotropic model, in which the scheme A, scheme B and scheme C are used as advection schemes respectively. The model uses 30 minutes as the advection time step and 7.5 minutes as the adjustment time step. This choice and other considerations will be described in section 5. Meanwhile, Fig. 7 shows the verifying 500 mb geopotential analysis fields. Comparing Fig. 4, 5, 6 with Fig. 7 , it is very clear that the intensities of synoptic systemes of 12-48 hour forecasts using scheme A are weaker than that forecasted by scheme B or scheme C. The forecasts using scheme B is very similar with that forecasted by using scheme C. We conclude that these results are caused by the fact that scheme A has lower ac-

curacy than the others, and that scheme B has the same accuracy as scheme C.

Fig. 8, 9 and Fig.10 show 12-48 hour forecasts of a barotropic model in which schemes A, B, C are used respectively. The difference between Fig.4, 5, 6 and Fig. 8, 9, 10 is the time step. Here the advection time step is 45 minutes and the adjustment time step is 11.25 minutes. These forecasts are very similar and show that noise appears in the subtropic area of all forecasts. Since in the scheme C the splitting technique was not used, this result indicates that truncation error is caused by choosing an unsuitable advection time step for splitting different physical processes. This means that when we use the splitting technique with a suitable time step in different physical processes of a barotropic model, using a downstream splitting advection scheme with one dimensional cubic spline interpolation does not differ much from using upstream advection scheme with the two dimensional cubic spline interpolation (nonsplitting scheme).

Table 1 shows the CPU time (on NAS 9000) of the advection part of a 48 hour forecast for different advection schemes. It indicates that scheme B uses only half as much CPU time as the upstream scheme. Since a vector operation can be used in scheme B, it can save much more computer time than the upstream scheme.

Table 1 CPU TIME (on NAS 9000, unit: second)

SCHEME	CPU TIME
Scheme A	44.1079
Scheme B	47.6701
Upstream Scheme	86.4311

Table 2 and Table 3 show the evolution of ratio of kinetic energy K, potential energy P and total energy (K+P) during 48 hour integrations using scheme B and scheme C as the advection scheme (with advection time step 30 minutes) respectively. K0 and P0 are the kinetic energy and potential energy at t=0.

Table 2 (splitting scheme)

Time of FCST	K/ /K0	P/ /P0	(K+P)/ /(K0+P0)
0	1	1	1
12	0.986591	0.998939	0.996771
24	0.983330	0.999340	0.996350
36	0.990215	0.998869	0.997351
48	0.979836	1.000204	0.996630

Both Tables show that the loss of about 0.015 of

kinetic energy during the first 12 hours quickly levels off. The potential energy remains constant and is very nearly conserved.

Table 3 (upstream scheme)

Time of FCST	K/ /K0	P/ /P0	(K+P)/ /(K0+P0)
0	1	1	1
12	0.984586	0.998926	0.996410
24	0.978577	0.999296	0.995660
36	0.982508	0.998809	0.995949
48	0.970113	1.000093	0.994834

3. THE ADJUSTMENT SCHEMES

In selecting an integration scheme for the adjustment process, we still put emphasis on high accuracy and less time consumption. In this paper a two time-level scheme, and a forward-backward scheme are compared.

A. Two-Time-Level Scheme

The two time-level scheme is, strictly speaking, a Eulerian scheme but is quite similar to a second-order advection scheme. We use " * " to mark the position of the point and the

value of variables at the first-time level. In the two-time level scheme u*, v* and ϕ^* are calculated at the same point which is shown in Fig. 11. The difference equations corresponding to that are

$$\begin{cases} u^*_{i+\frac{1}{2},j+\frac{1}{2}} = \overline{u^n}^{xy}_{i+\frac{1}{2},j+\frac{1}{2}} - \left(\overline{m\,\overline{\phi^n_x}^{\,y}}\right)_{i+\frac{1}{2},j+\frac{1}{2}} \Delta t/2 + \left(\overline{f\,\overline{v^n}^{xy}}\right)_{i+\frac{1}{2},j+\frac{1}{2}} \Delta t/2 \\ v^*_{i+\frac{1}{2},j+\frac{1}{2}} = \overline{v^n}^{xy}_{i+\frac{1}{2},j+\frac{1}{2}} - \left(\overline{m\,\overline{\phi^n_y}^{\,x}}\right)_{i+\frac{1}{2},j+\frac{1}{2}} \Delta t/2 - \left(\overline{f\,\overline{u^n}^{xy}}\right)_{i+\frac{1}{2},j+\frac{1}{2}} \Delta t/2 \\ \phi^*_{i+\frac{1}{2},j+\frac{1}{2}} = \overline{\phi^n}^{xy}_{i+\frac{1}{2},j+\frac{1}{2}} - \left(\overline{m\,\overline{\phi^n}^{xy}}\right)_{i+\frac{1}{2},j+\frac{1}{2}} \Delta t/2 \left[\left(\overline{\frac{u^n}{m}}^{\,y}\right)_x + \left(\overline{\frac{v^n}{m}}^{\,x}\right)_y \right]_{i+\frac{1}{2},j+\frac{1}{2}} \end{cases} \tag{10.a}$$

and

$$\begin{cases} u^{n+1}_{i,j} = u^n_{i,j} - \left(m\,\overline{\phi^*_x}^{\,y}\right)_{i,j} \Delta t + \left(f\,\overline{v^*}^{xy}\right)_{i,j} \Delta t \\ v^{n+1}_{i,j} = v^n_{i,j} - \left(m\,\overline{\phi^*_y}^{\,x}\right)_{i,j} \Delta t - \left(f\,\overline{u^*}^{xy}\right)_{i,j} \Delta t \\ \phi^{n+1}_{i,j} = \phi^n_{i,j} - \left(m\,\overline{\phi^*}^{xy}\right)_{i,j} \Delta t \left[\left(\overline{\frac{u^*}{m}}^{\,y}\right)_x + \left(\overline{\frac{v^*}{m}}^{\,x}\right)_y \right] \end{cases} \tag{10.b}$$

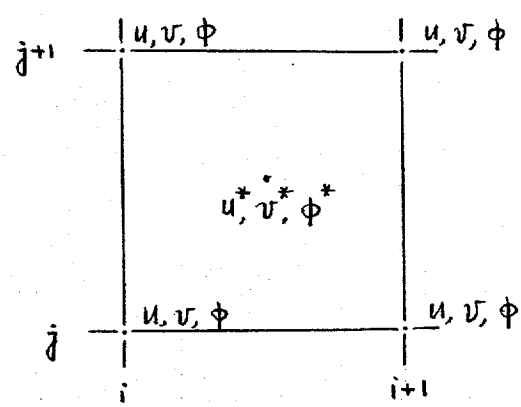

Fig. 11 the distribution of calculated points at the 1st and 2nd time levels

where A^n , A^*, A^{n+1} are the value of variable A at the time $n\Delta t$, $(n+1/2)\Delta t$, and $(n+1)\Delta t$ respectively; $\overline{A}^x$, $\overline{A}^y$, $\overline{A}^{xy}$ show that the average value of variable A is at x-direction, y-direction, and both directions respectively; A_x (A_y) is the difference of variable A at x direction (y direction); Δt is time step; m is the map factor and $f = 2\Omega \sin\varphi$

B. Foreward-Backward Adjustment Scheme

Since the foreward-backward scheme allows a time step which is twice as long as that of the leapfrog scheme and is very convenient, many authors use this technique for the adjustment scheme in their models (Gadd, 1978; Bates and McDonald, 1982). The difference equations are

$$\begin{cases} \Phi_{i,j}^{n+1} = \Phi_{i,j}^{n} - \Delta t \, (m^2 \Phi^n)_{i,j} \left[\left(\frac{u^n}{m}\right)_x + \left(\frac{v^n}{m}\right)_y \right]_{i,j} \\ u_{i,j}^{n+1} = u_{i,j}^{n} - \Delta t \, (m \, \Phi_x^{n+1})_{i,j} + (f \, v^n)_{i,j} \Delta t \\ v_{i,j}^{n+1} = v_{i,j}^{n} - \Delta t \, (m \, \Phi_y^{n+1})_{i,j} - (f \, u^n)_{i,j} \Delta t \end{cases} \qquad (11.a)$$

If we want to obtain a stronger dissipation the following equations can be used,

$$
\begin{cases}
\phi^{n+1}_{i,j} = \phi^{n}_{i,j} - \Delta t \; (m^2 \phi^{n})_{i,j} \; [\; (\frac{u^n}{m})_x + (\frac{v^n}{m})_y \;]_{i,j} \\
u^{n}_{i,j} = u^{n}_{i,j} - \Delta t (m \phi^{n+1}_{x})_{i,j} + f \; [\; v^{n}_{i,j} - \Delta t \; (m \phi^{n}_{y} + f \, u^{n})_{i,j} \; /2 \;] \Delta t \\
v^{n}_{i,j} = v^{n}_{i,j} - \Delta t (m \phi^{n+1}_{y})_{i,j} - f \; [\; u^{n}_{i,j} - \Delta t \; (m \phi^{n}_{x} - f \, v^{n})_{i,j} \; /2 \;] \Delta t
\end{cases}
\tag{11.b}
$$

Table 4 and Table 2 show the evolution of ratio of kinetic energy, potential energy and total energy during the 48 hour integration using the two-time level adjustment scheme and the forward-backward adjustment scheme respectively, where K0 and P0 are the kinetic energy and potential energy at t=0.

Table 4 (two-time level scheme)

Time of FCST	K/ /K0	P/ /P0	(K+P)/ /(K0+P0)
0	1	1	1
12	0.985673	0.998827	0.996519
24	0.983074	0.999290	0.996444
36	0.988170	0.998781	0.996919
48	0.976647	1.000163	0.996037

Both Tables show their evolutions of energy ratio are very similar each other. But the CPU time of adjustment using the above two schemes for a 48 hour forecast is different (Table 5).

Obviously, using the forward-backward scheme can save computer time.

Table 5 CPU time (on NAS 9000, unit: second)

SCHEME	CPU TIME
Two-time level	76.6221
Forward-backward	33.1701

4. THE BOUNDARY CONDITION

In this section we'll compare briefly two boundary condition schemes: Scheme A -- inflow/outflow boundary condition; Scheme B -- boundary values come from outside. The boundary zone

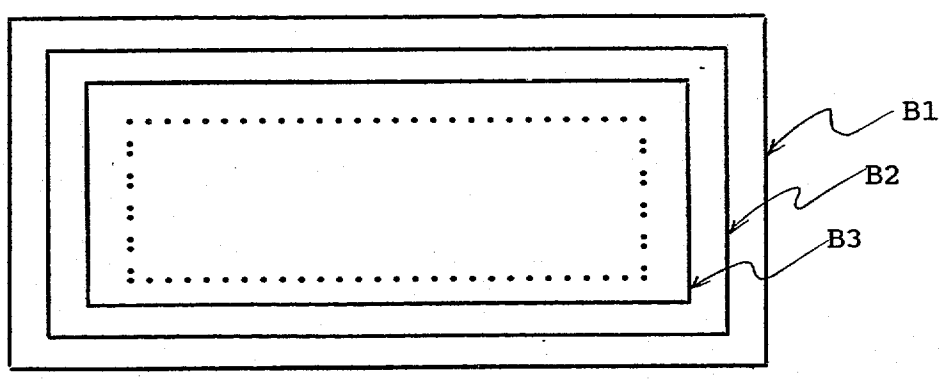

Fig. 12 the boundary zone

is shown in Fig. 12. The boundary values at boundary points

which lie on B1 would be determined by the above two schemes respectively. The values at points of the buffer zone (B2 and B3) are obtained by solving the Laplace equation $\nabla^2 A = 0$ using relaxation method with outer boundary conditions at boundary B1 and internal boundary conditions. In our experiment, the inflow/outflow boundary condition means that the data at the inflow grid points of B1 come from the analysis field and the data at the outflow grid points of B1 are determined by a quasi-Lagrangian method as a linear advection scheme. After introducing the above boundary conditions, there was a strong discontinuity between the inflow point and the outflow point so that the three point smoothing operator is used along boundary of B1. The scheme B means that all boundary values at points of B1 come from the analysis field.

The results of a 48 hour integration with scheme A and scheme B are shown in Fig.13 and Fig. 5 respectively. The split advection scheme and forward-backward adjustment scheme are used in both integrations. In Fig. 13 it is clear that there is very strong gradient of height in the northeast boundary zone. However, in Fig. 5 there is neither a discontinuity nor an abnormal gradient of height on the boundary zone. This very strong gradient of height in the inflow/outflow boundary scheme is caused by

introducing boundary data from the linear advection field at outflow point of B1.

5. INTEGRATION OF THE BAROTROPIC MODEL

Following the above experiments, we chose the splitting advection scheme described in section 2, the foreward-backward adjustment scheme and a boundary condition scheme B with a buffer zone. The model was integrated on the LFM grid, a polar stereographic projection. The grid interval is 190.5 km at 60 N. The time step for advection scheme is 30 minutes (or 45 minutes), and for the adjustment scheme is 7.5 minutes (or 11.25 minutes). After each four step integrations of the adjustment process by forward-backward scheme, one step integration of the advection by quasi-Lagrangian method is made.

Initialization data for wind and heights are taken from the LFM. When a longer advection time step, for example 45 minutes or one hour, is used, a 5-point smoothing over the interior part of the grid is needed each 6 hours to inhibit the growth of small turbulence. Forecast results after 12 - 48 hour integration (start time: 0000 GMT, Oct. 10, 1986) by this barotropic model are shown on Fig. 5 and Fig. 14 respectively.

Comparing these results, we can see clearly that both

prediction results have good agreement on the positions and intensities of main weather systems. This means either the advection scheme of this barotropic model is suitable or the advection part in the atmosphere is most important. The main difference between the two forecasts and analysis fields are the trough located near 100-110°W and the High center upstream of this trough: (1) After 24 hours, the trough forecast is weaker than that of the analysis field. This was probably caused by a strong baroclinic property of the trough. (2) The tendency of the High forecasted by the barotropic model was not in agreement with that on the analysis fields. (3) The forecast in the subtropical zone is consistent with the analysis field very much. This may be caused by accuracy of advection scheme and boundary conditions or by barotropic property at the subtropical area.

6. SUMMARY

In this model we used the split quasi-Lagrangian method in order to avoid losing advection scheme accuracy caused by incorrect position of the interpolated point. Although a splitting scheme leads to a new truncation error, this error is not significant when we chose a suitable time step. The experiments

with different advection schemes (splitting scheme and nonsplitting scheme) show that most truncation error is caused by using splitting technique in different physical processes of the barotropic model. After a splitting scheme with a suitable advection time step is used in different physical processes, the downstream - splitting advection scheme with the cubic spline interpolation has a high accuracy which is approximate to that of a upstream-nonsplitting advection scheme with the two dimensional cubic spline interpolation. But the former can save much more computer time. Since the limitation of time step is needed for a barotropic model, in which the splitting technique was used in different physical processes, the advection time step cannot be too long. Otherwise the truncation error will become very significant. This limitation of time step causes a slight loss in the advantages of the quasi-Lagrangian method. But using the splitting technique in physical processes provides an opportunity to use a downstream splitting advection scheme with one dimensional cubic spline interpolation; therefore, it is possible to use a vector operation and still have the 4th order advection accuracy.

The results of two boundary condition schemes indicated that for a regional model with a one-way boundary conditions it

is important to provide accurate boundary conditions.

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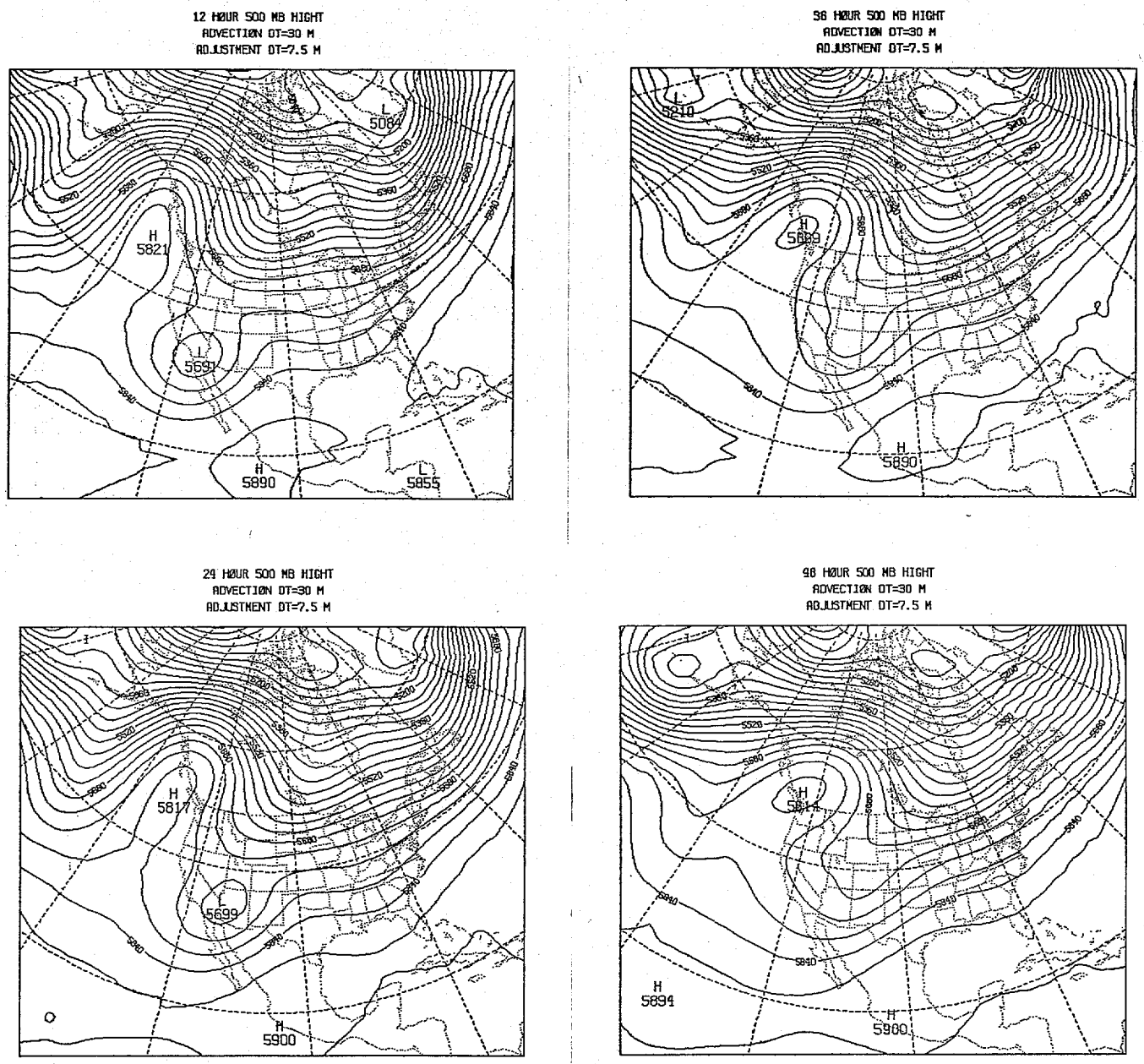

Fig. 4 12 - 48 hour forecasts of height field
start time: 00 GMT, Oct. 10, 1986
advection scheme A
forward-backward adjustment scheme
Tadv = 30 minutes
Tadj = 7.5 minutes

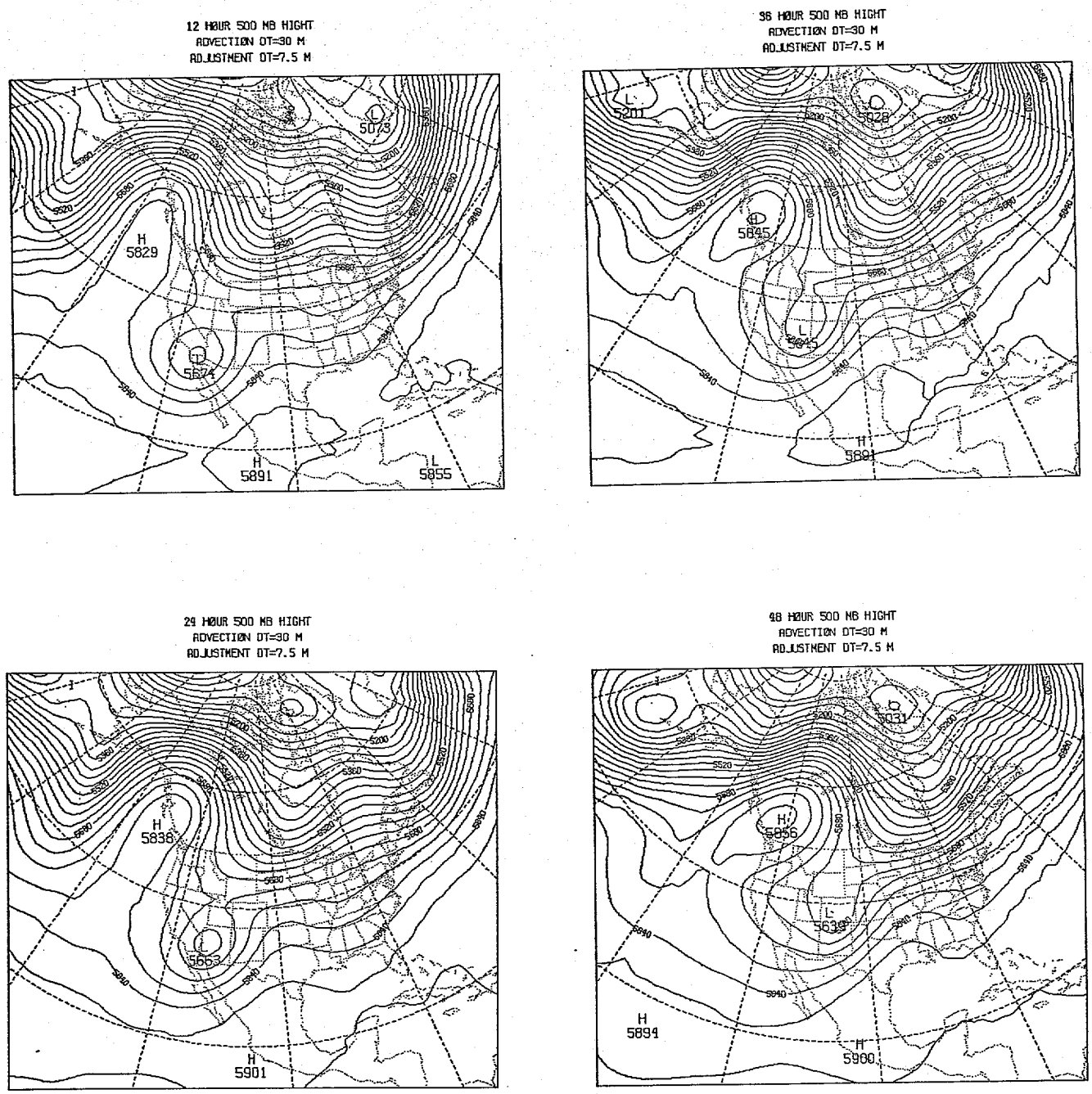

Fig. 5 12 - 48 hour forecasts of height field
start time: 00 GMT, Oct. 10, 1986
advection scheme B (splitting)
forward-backward adjustment scheme
Tadv = 30 minutes
Tadj = 7.5 minutes

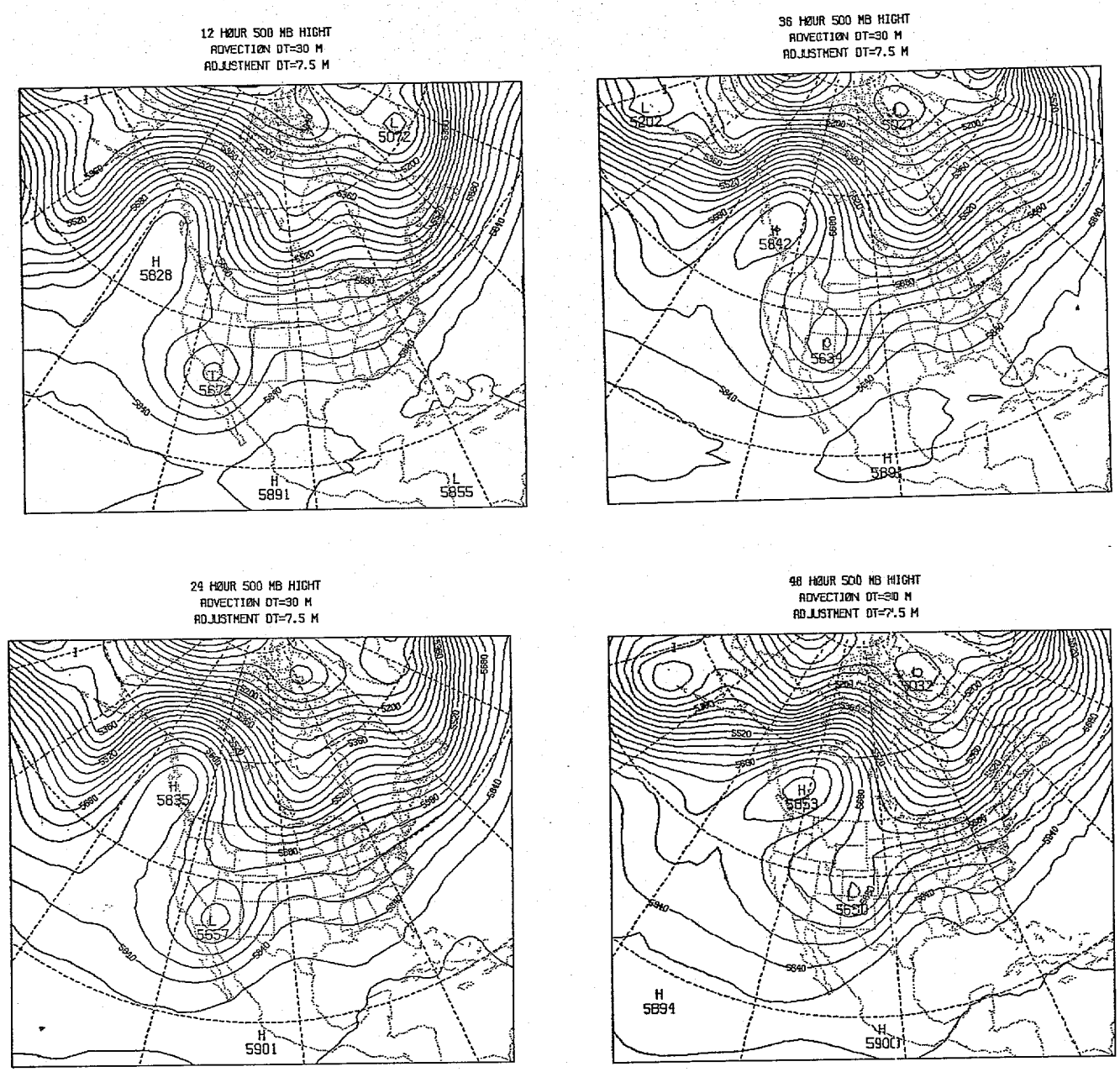

Fig. 6 12 - 48 hour forecasts of height field
start time: 00 GMT, Oct. 10, 1986
advection scheme C (upstream)
forward-backward adjustment scheme
Tadv = 30 minutes
Tadj = 7.5 minutes

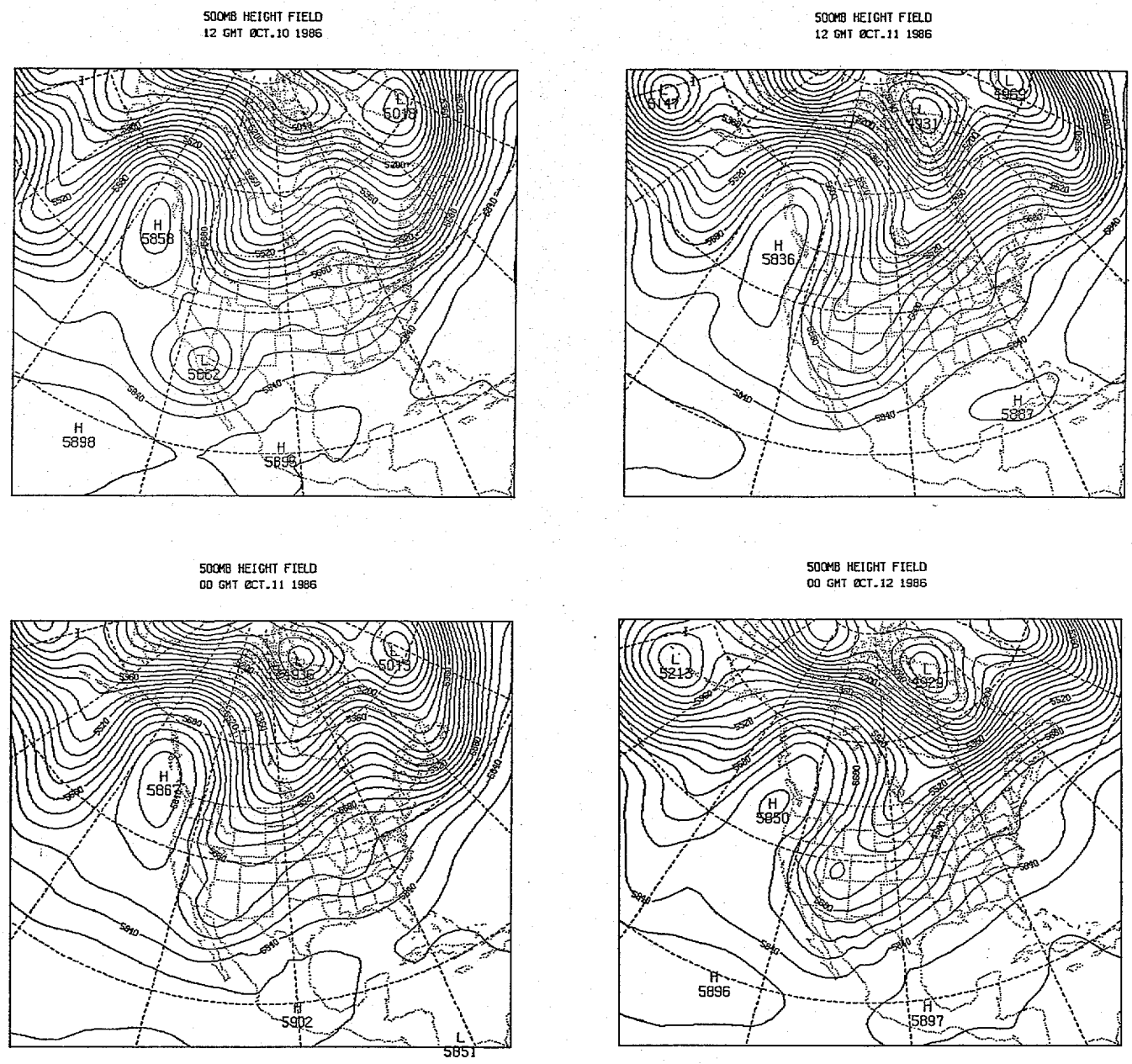

Fig. 7 Analysis fields of 500 mb geopotential corresponding Fig. 4, 5, and 6

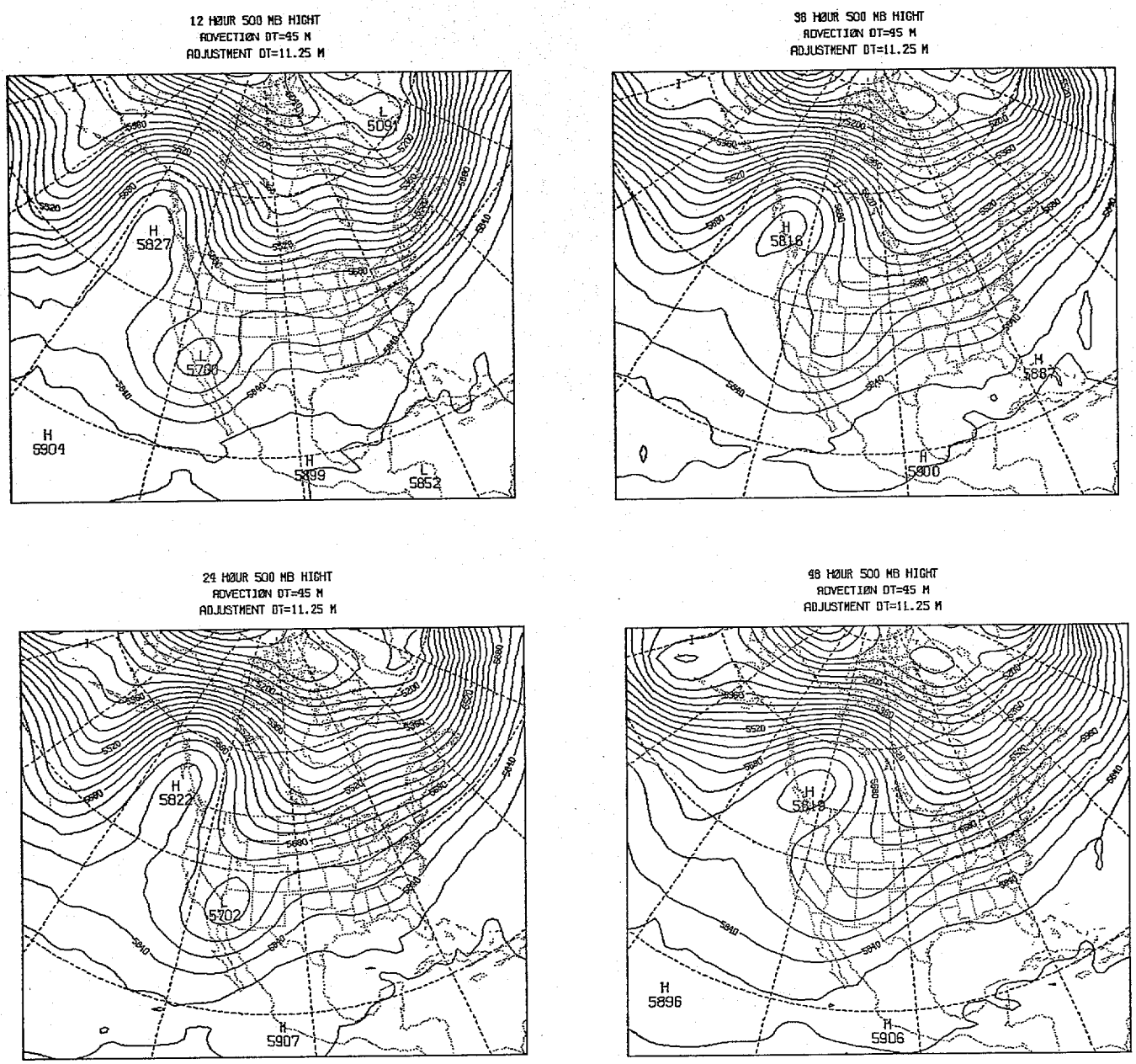

Fig. 8 12 - 24 hour forecasts of height field
start time: 00 GMT, Oct. 10, 1986
advection scheme A
forward-backward adjustment scheme
Tadv = 45 minutes
Tadj = 11.25 minutes

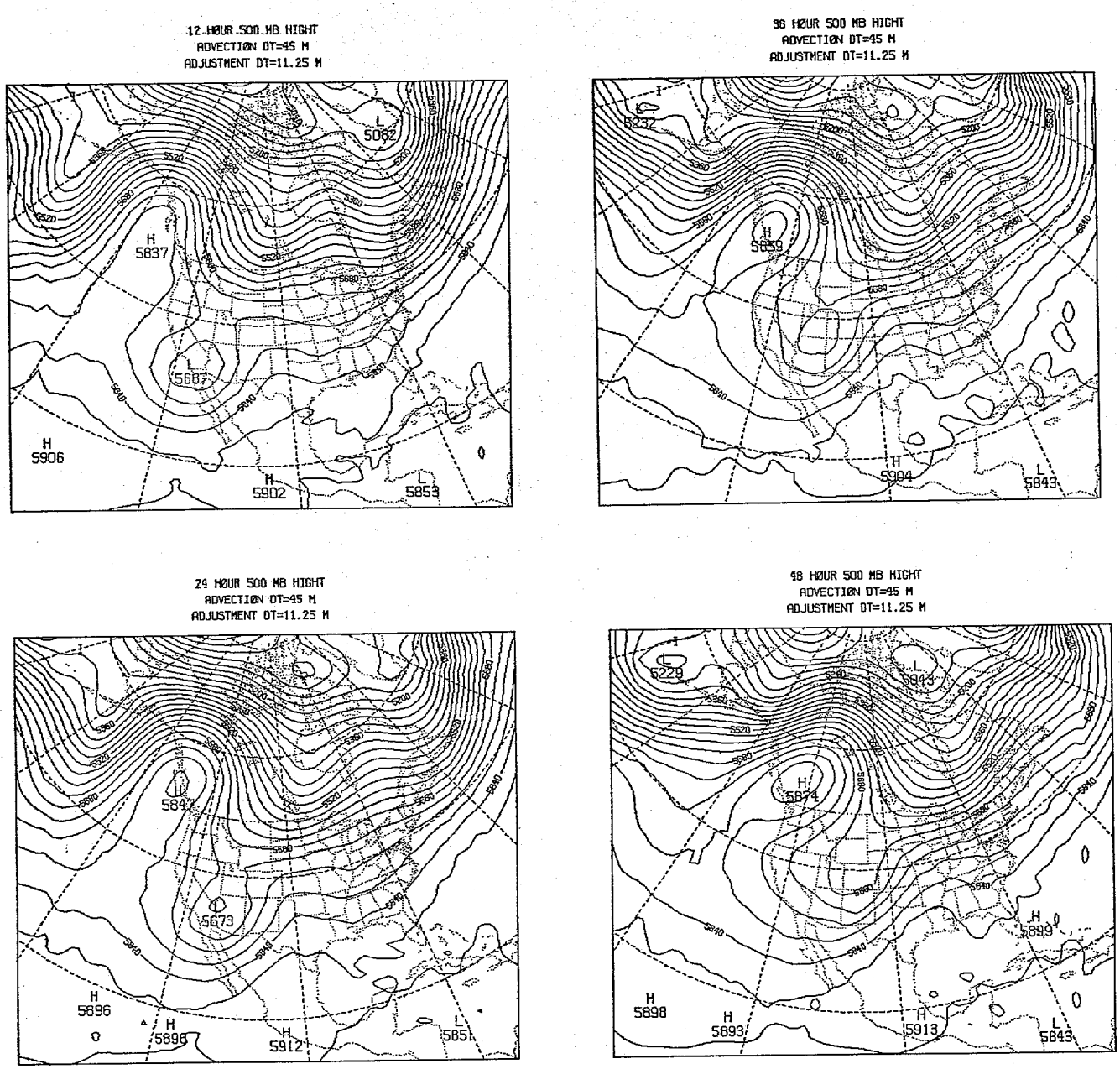

Fig. 9 12 - 48 hour forecasts of height field
start time: 00 GMT, Oct. 10, 1986
splitting advection scheme (B)
forward-backward adjustment scheme
Tadv = 45 minutes
Tadj = 11.25 minutes

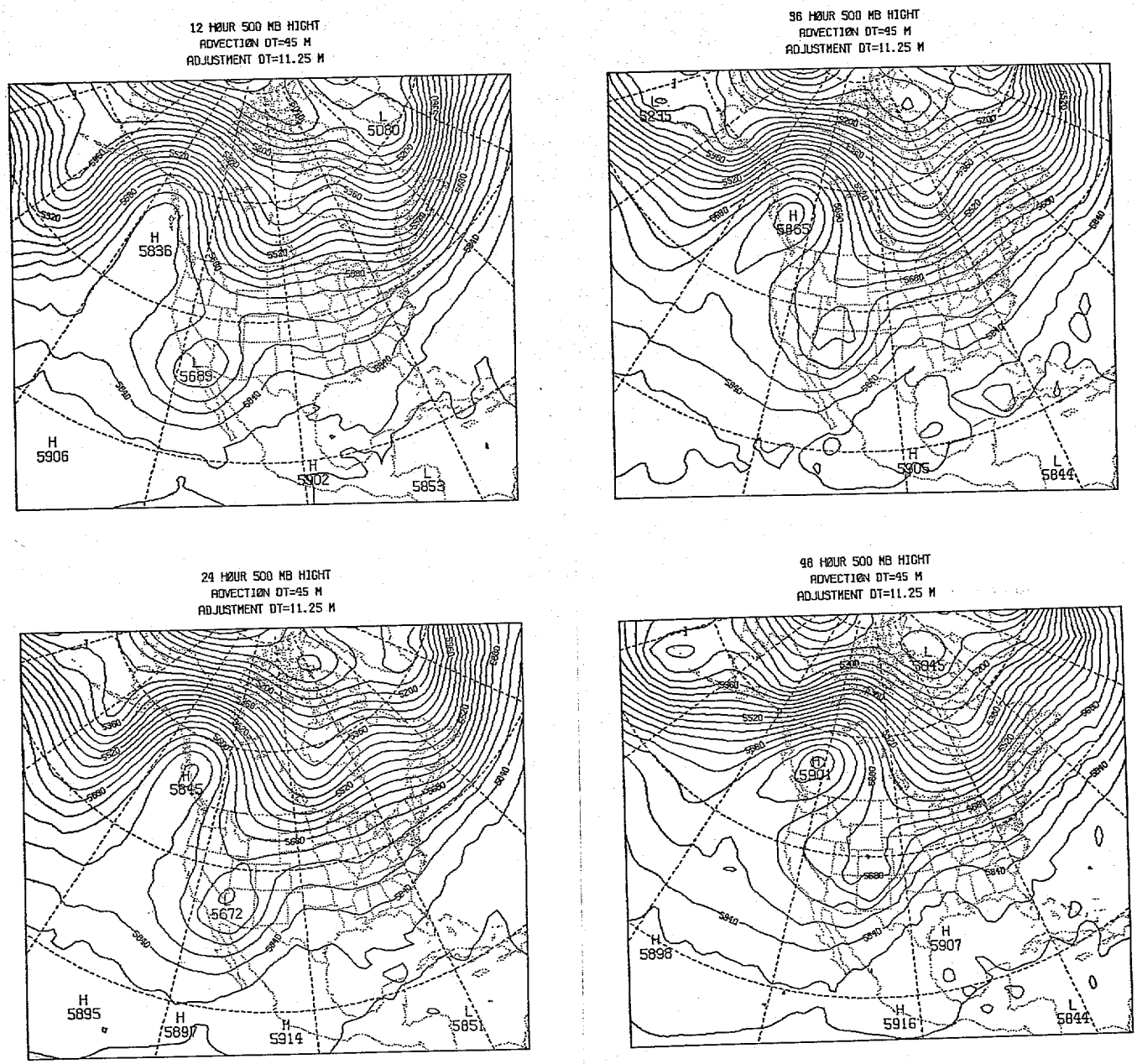

Fig. 10 12 - 48 hour forecasts of height field
start time: 00 GMT, Oct. 10, 1986
upstream advection scheme
forward-backward adjustment scheme
Tadv = 45 minutes
Tadj = 11.25 minutes

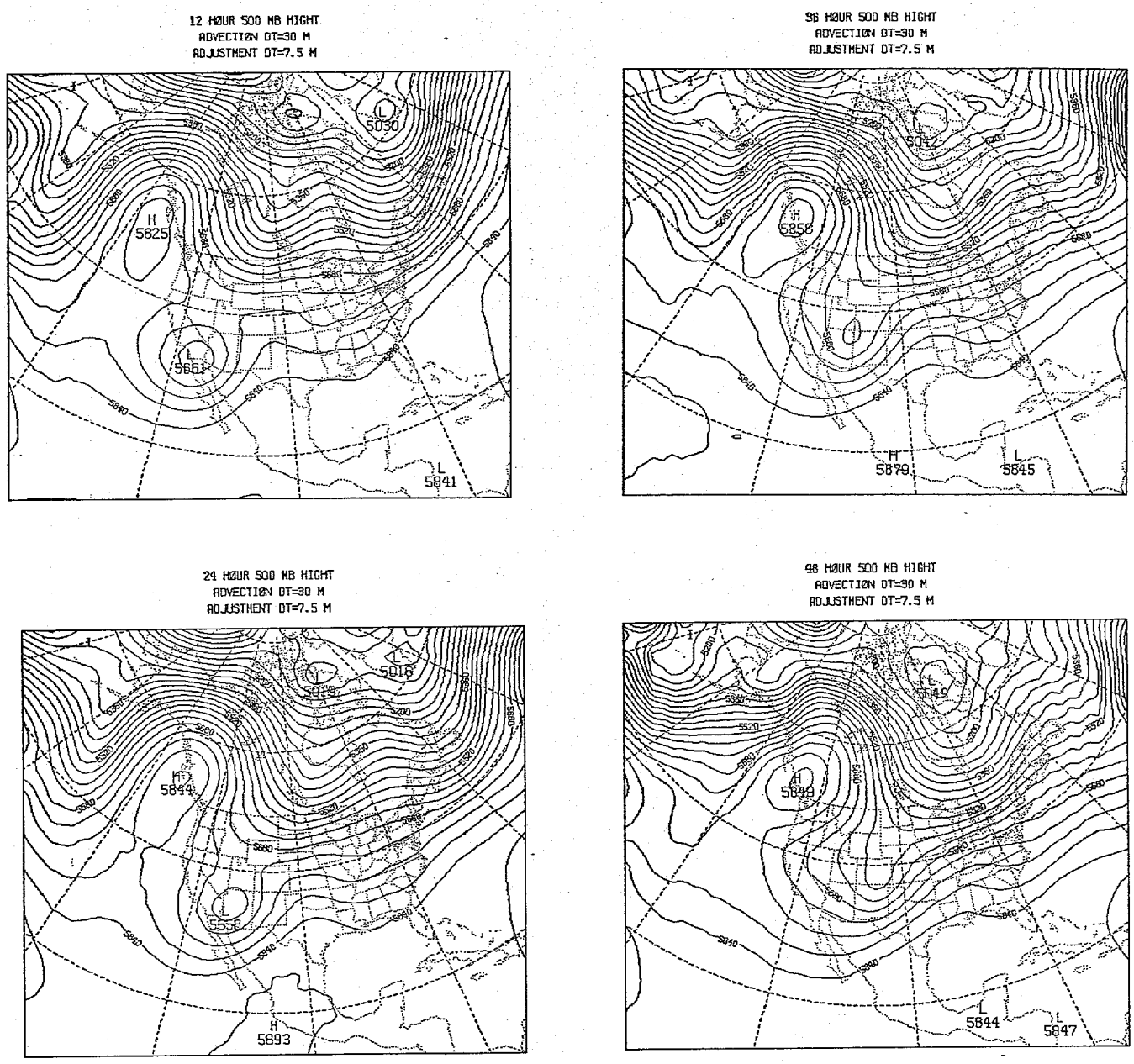

Fig. 13 12 - 48 hour forecasts of height field as the same with Fig. 5 but inflow/ outflow boundary condition scheme

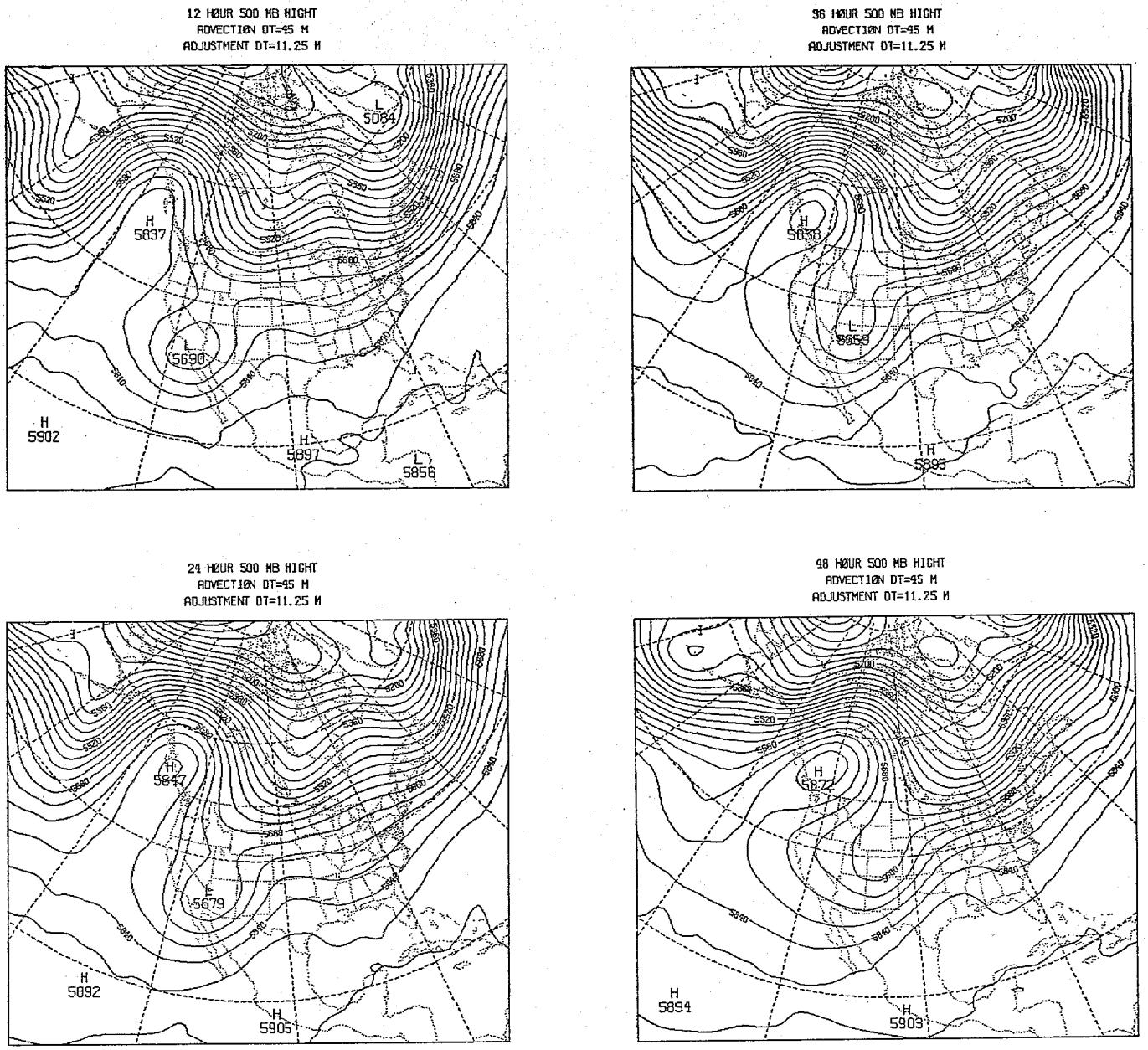

Fig. 14 12 - 48 hour forecasts of height field as the same with Fig. 9 but using one 5-point smoothing each 6 hours.