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AN ECONOMICAL AND COMPATIBLE SCHEME FOR PARAMETERIZING THE STABLE SURFACE LAYER IN THE MEDIUM RANGE FORECAST MODEL

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ABSTRACT

A new method for parameterizing turbulent fluxes of momentum, heat, and moisture within the stable surface layer is presented. The method is designed to supplant the existing scheme used in the Geophysical Fluid Dynamics Laboratory's 'E2' physics package. The existing E2 scheme is incompatible with the turbulent transfer coefficients that are used in the current version of the Medium Range Forecast Model (the MRF86). The new method unifies the subdivided stable regime of the E2 physics and provides a compatible, economical, and noniterative technique that does not possess the limitation of a finite critical bulk Richardson number.

Introduction

A technique is presented for computing the surface layer fluxes of momentum, heat, and moisture during stable conditions. The method is intended to replace the existing scheme used in the Geophysical Fluid Dynamics Laboratory's 'E2' boundary layer parameterization.

The 'E2' method divides the stable surface layer into four regimes: weakly stable, mildly stable, strongly stable, and nonturbulent. Except for the nonturbulent category, each regime has a specific flux-profile relation. These relations are functions of the nondimensional height, $\xi \equiv Z/L$, in which L is the Obukhov length. The first three regimes are specified by, $\varphi_M(\xi) = \varphi_H(\xi) = \varphi_Q(\xi)$, in which

weakly stable: $$\varphi_M(\xi) = 1 + 5\xi \qquad 0 \leq \xi \leq 0.5 \tag{1.1}$$

mildly stable: $$\varphi_M(\xi) = 8 - \frac{4.25}{\xi} + \frac{1}{\xi^2} \qquad 0.5 \leq \xi \leq 10 \tag{1.2}$$

strongly stable: $$\varphi_M(\xi) = 0.76 \qquad 10 \leq \xi < \infty. \tag{1.3}$$

The functions $\varphi_{M,H,Q}$ are the universal nondimensional vertical shears of windspeed, potential temperature, and specific humidity and are defined by,

$$\varphi_M(Z/L) = \frac{kZ}{u_*}\frac{\partial U}{\partial Z}, \tag{1.4}$$

$$\varphi_H(z/L) = \frac{kz}{b\theta_*}\frac{\partial\theta}{\partial z}, \tag{1.5}$$

$$\varphi_Q(z/L) = \frac{kz}{cq_*}\frac{\partial q}{\partial z}, \tag{1.6}$$

in which $k(\doteq 0.4)$ is the von Kármán constant, and u_*, θ_*, and q_* are the turbulent scaling parameters for the fluxes of momentum, heat, and humidity. The constants b and c are on the order of unity for all published flux-profile relations. For the 'E2' profiles, b = c = 1.

It is fairly common modeling practice to use (1.1) for the entire stable regime $\xi \geq 0$. The use of (1.1) for all $\xi \geq 0$ leads to a critical gradient Richardson number ($\equiv R_c$) of 0.20. Modeled surface layer turbulent fluxes vanish as the critical Richardson number is approached. As the fluxes vanish, the nondimensional length diverges ($\xi \rightarrow +\infty$), and the Obukhov length approaches zero.

The most significant effect of introducing relations (1.1-3), as suggested by Carson and Richards (1977), is to shift the critical Richardson number from 0.2 to 1.32. This shift permits the existence of modeled turbulent fluxes over a more generous range of stability than would be allowed by (1.1). The turbulent fluxes of momentum, heat, and moisture in the surface layer are denoted by F_M, F_H, F_Q and are given by,

$$F_M = -\rho u_*^2 = -\rho(kz)^2 \varphi_M^{-2}\left(\frac{\partial U}{\partial z}\right)^2 \tag{1.7}$$

$$F_H = -\rho C_p u_* \theta_* = -\rho C_p (kz)^2 (\varphi_M \varphi_H)^{-1} \frac{\partial U}{\partial z} \frac{\partial \theta}{\partial z} \quad (1.8)$$

$$F_Q = -\rho \mathcal{L} u_* q_* = -\rho (kz)^2 \mathcal{L} (\varphi_M \varphi_Q)^{-1} \frac{\partial U}{\partial z} \frac{\partial q}{\partial z}. \quad (1.9)$$

In this note we shall make the standard assumption that $\varphi_Q = \varphi_H$, although fully convincing corroborative evidence is lacking. We shall also assume that $\varphi_M = \varphi_Q$ during stable conditions. For unstable conditions it is well known (see, for example, Businger et al., 1971) that this is not the case, but for mildly stable conditions, experimental data suggest that $\varphi_M = \varphi_H = \varphi_Q$. For increasing stability, it is possible that φ_M and φ_H begin to differ substantially (Turner, 1973; Kondo et al. 1977), but we shall not attempt to deal with this possibility in this note.

To help demonstrate the need for an alternative flux-profile formulation, we first show that for modeling purposes, standard linear $\varphi_{M,H}$ of the form

$$\varphi_M = \beta_M (1 + \alpha_M \xi) \quad (1.10)$$

$$\varphi_H = \beta_H (1 + \alpha_H \xi) \; ; \; \alpha_M \sim \alpha_H \; ; \; \beta_M \sim \beta_H \quad (1.11)$$

can impose an undesirable restriction. From the definition of the Obukhov length (Businger et al., 1971),

$$L \equiv \frac{\theta}{kg} \frac{u_*^2}{\theta_*} \quad (1.13)$$

and from (1.4,5), we see that the connecting relation between the gradient Richardson number $\left[\equiv \frac{g}{\theta}\frac{\partial\theta}{\partial z}\left(\frac{\partial U}{\partial z}\right)^{-2}\right]$ and the nondimensional height ξ is simply

$$\xi = \frac{\varphi_M^2}{\varphi_H} R, \quad b = 1. \tag{1.14}$$

For the linear $\varphi_{M,H}$ in (1.10,11), the relation between $\varphi_{M,H}$ and R is

$$\xi = \frac{\beta_M^2}{\beta_H}(1+\alpha_M \xi)^2 (1+\alpha_H \xi)^{-1} R \tag{1.15}$$

As $\xi \to \infty$, (1.15) shows that R approaches a finite limit $(\equiv R_c)$ that is given by

$$R_c = \alpha_H \beta_H (\alpha_M \beta_M)^{-2} \tag{1.16}$$

Field experiments indicate that $\beta \cong 0.74 - 1$ and $\alpha \cong 4 - 7$. The data of Businger et al. (1971) yield $R_c = 0.213$, while the data of Dyer and Hicks (see Dyer, 1974) yield $R_c = 0.20$. Both values of are probably too restrictive for numerical models in which R quite commonly exceeds R_c.

The main consequence of a low R_c is that too much of the upward daytime heat from the air-ground interface into surface layer and the layers above the surface layer cannot return to the ground during stable nocturnal conditions. A relaxation of the R_c restriction is needed to enhance the downward flux at night. Compared to the daytime convective surface layer, the nocturnal surface layer is shallow with small, patchy

turbulent eddies mixed with gravity waves. As Yu (1978) has pointed out, there is no commonly agreed upon definition of the depth of the stable boundary layer, much less a simple mathematical description. Wyngaard (1985) observed of the stable boundary layer that, "..its energetics can be in a delicate and precarious balance, and extremely sensitive to changes in the mean wind profile (whose shear is its energy source) and the mean temperature profile (whose stable lapse rate severely limits its vertical motions)".

The lack of clear-cut knowledge of the stable boundary layer's profile laws permits some constructive tampering, as in Carson and Richards' profile laws (1.1-3). Their expression for mild stability (1.2) does not arise from empirical data, but is merely a smooth interpolation from (1.1) to (1.3). Only (1.1) has strong experimental support.

II. The extension of R_c and a new formulation for $\varphi_{M,H}$

In this section we shall propose a simple formulation for $\varphi_{M,H}$ that exhibits neither a critical gradient nor a critical bulk Richardson number. In addition, the formulation does not require three partitioning relations as is the case with (1.1-3).

To motivate the new form for $\varphi_{M,H}$, we first examine the behavior of ζ and the turbulent diffusion coefficients for the general linear profiles (1.10,11). As shown in (1.15), a quadratic equation must be solved to compute ζ from a given gradient Richardson number. A simpler, more perspicuous approximation, valid for $\alpha_{M,H}\,\zeta >> 1$, results from expressing (1.15) as

$$\xi = \frac{\xi}{R_c}\left(1+\frac{1}{\alpha_M \xi}\right)^2\left(1+\frac{1}{\alpha_H \xi}\right)^{-1} R$$

and using

$$(1+\epsilon)^r = 1 + r\epsilon + O(\epsilon^2)$$

to get

$$\xi = \gamma R/(1-R/R_c) \tag{2.1}$$

in which

$$\gamma = \frac{2\alpha_H - \alpha_M}{\alpha_M \alpha_H R_c} . \tag{2.2}$$

The expressions for the diffusion coefficients $K_{M,H}$ follow from (1.7,8) and are given by

$$K_{M,H} = \ell^2 S_{M,H}(\xi) \cdot \frac{\partial U}{\partial Z} \tag{2.3}$$

in which

$$S_{M,H} = \left[\beta_{M,H}\beta_M(1+\alpha_{M,H})(1+\alpha_M)\right]^{-1}, \tag{2.4}$$

and the mixing length ℓ is equal to kZ. For the special case of the Dyer-Hicks profile (1.1), we have

$$\xi = \frac{R}{1-\alpha R} \; ; \; R < \alpha^{-1} \tag{2.5}$$

and

$$K_{M,H} = \ell^2 \frac{\partial U}{\partial Z}(1-\alpha R)^2 \; ; \; R < \alpha^{-1} \tag{2.6}$$

in which $\alpha = R_c^{-1} = 5.0$. The turbulent diffusion coefficients drop to zero rapidly and ζ diverges as R approaches 0.2, a rather stringent stability range.

This undesirable situation is substantially offset by invoking profile laws (1.2) and (1.3). Eq. (1.2) requires that a cubic equation be solved to compute ζ from R. By substitution, we see that $R \rightarrow 1.32$ as $\zeta \rightarrow 10$. The critical gradient Richardson number also equals 1.32 since, for $\zeta \geq 10$, R is independent of ζ and equals $(0.76)^{-1}$, which is just 1.32. Thus, with the Carson and Richards' profile laws, the range of the gradient Richardson numbers is expanded by a factor of 6-7 over the simpler linear $\varphi_{M,H}$.

In calculating the surface fluxes in numerical models, it is the bulk rather than gradient Richardson number that is the useful nondimensional parameter. Given the potential temperature difference $\Delta\Theta$ between z_0 ($\equiv$ roughness height) and z (frequently taken as the height of the first model layer) and the windspeed U at z, then a bulk Richardson number, defined as $R_B \equiv \frac{gz}{\Theta} U^{-2} \Delta\Theta$ can be shown to be related to Z/L by an expression analogous to (1.14), that is,

$$\frac{Z}{L} = \frac{F_M^2}{F_H} R_B \tag{2.7}$$

in which

$$F_{M,H} = \int_{z_0}^{z} \frac{dz'}{z'} \varphi_{M,H}(z'/L) \tag{2.8}$$

For the Carson and Richards profiles, $F_{M,H}$ are given by

$$F^{I}_{M,H} = \int_{\xi_0}^{\xi} \frac{d\xi}{\xi} \varphi^{I}_{M,H} = \ln \frac{\xi}{\xi_0} + 5(\xi - \xi_0) \text{ for } 0 \leq \xi \leq 0.5 \tag{2.9}$$

$$F^{II}_{M,H} = F^{I}_{M,H}(\xi = 0.5) + \int_{\xi'=0.5}^{\xi} \frac{d\xi'}{\xi'} (8 - 4.25\,\xi'^{-1} + \xi'^{-2}) \tag{2.10}$$

for $0.5 \leq \xi \leq 10$,

and

$$F^{III}_{M,H} = F^{II}_{M,H}(\xi = 10) + \int_{\xi=10}^{\xi} \frac{d\xi'}{\xi'} (0.76\,\xi'), \quad \xi \geq 10. \tag{2.11}$$

In the case of a weakly stable surface layer ($\xi \leq$ 0.5), we have

$$F_{M,H} = \ln \frac{z}{z_0} + 5(\xi - \xi_0)$$

or (for $\xi_0 \ll 1$, which is generally the case),

$$\xi \doteq \frac{R_B \ln(z/z_0)}{1 - 5R_B} \tag{2.12}$$

The mildly and strongly stable cases are nonlinear and can be solved by iteration; however, from (2.11) it is evident that the critical R_B is equal to $(0.76)^{-1} = 1.32$. This is the same value as the critical gradient Richardson number. For $R_B \rightarrow 1.32$, ξ diverges and the surface fluxes vanish.

A new flux-profile expression can be devised by first noting that (2.6),

$$K_{M,H} = l^2 \frac{\partial U}{\partial z} (1-\alpha R)^2, \quad \alpha R \leq 1$$

can be approximated for $\alpha R \ll 1$ by

$$K_{M,H} = l^2 \frac{\partial U}{\partial z} (1+\alpha R)^{-2}. \tag{2.13}$$

Since $(1+\alpha R)^{-2} = 1 - 2\alpha R + O(R^2)$ and $(1-\alpha R)^2 = 1 - 2\alpha R + \alpha^2 R^2$ agree to first-order in R, it is reasonable to speculate that (2.6), which is valid only for $0 \leq R < \alpha^{-1}$, could be profitably supplanted by $(1+\alpha R)^{-2}$, which is valid for all $R \geq 0$. With $(1+\alpha R)^{-2}$, there is no critical gradient Richardson number at which surface fluxes vanish. Moreover, (2.13) is currently used at all levels above the surface layer in the new version of the Medium Range Forecast Model (the so-called MRF 86).

Adopting (2.13) as the formulation of choice for the surface layer means abandoning the Carson and Richards flux-profile expressions for

$\varphi_{M,H}$. Since

$$S_{M,H}(R) = \left[\varphi_M(\xi)\,\varphi_{M,H}(\xi)\right]^{-1},$$

it follows from (2.13) that the new $\varphi_{M,H}$ become

$$\varphi_M = \varphi_H = (1+\alpha R) \tag{2.14}$$

$$\xi = (1+\alpha R)R \tag{2.15}$$

or

$$R = \frac{-1 + \sqrt{1+4\alpha\xi}}{2\alpha} \tag{2.16}$$

and

$$\varphi_M = \varphi_H = \frac{1}{2}\left(1+\sqrt{1+4\alpha\xi}\right). \tag{2.17}$$

For small values of R and ξ , we have the usual linear results,

$$\xi \doteq R \tag{2.18}$$

$$\varphi_{M,H} = 1+\alpha\xi \tag{2.19}$$

but for large ξ , we have the results

$$\xi \sim \alpha R^2, \tag{2.20}$$

$$\varphi_{M,H} \sim (\alpha\,\xi)^{1/2}. \tag{2.21}$$

It is the departure from linearity in (2.20,1) that removes the possibility of a finite positive critical Richardson number.

III. Solutions for ξ as functions of R_B

In this section we show how ξ can be computed by iteration as a function of the bulk Richardson number. We shall also show that ξ can be approximated to acceptable accuracy without iteration. In either case, knowledge of ξ leads directly to u_*, θ_*, and q_*, and, therefore, to the turbulent fluxes $\mathcal{F}_{M,H,Q}$. We can also compute C_M and C_H, the bulk transfer coefficients for momentum and heat, from

$$C_M = k^2 / F_M^2 \tag{3.1}$$

$$C_H = k^2 / F_M F_H . \tag{3.2}$$

The fluxes $\mathcal{F}_{M,H,Q}$ are calculated from

$$\mathcal{F}_M = -\rho C_M U^2 \tag{3.3}$$

$$\mathcal{F}_H = -\rho C_\xi C_H U \Delta\theta \tag{3.4}$$

$$\mathcal{F}_Q = -\rho \mathcal{L} C_H U \Delta q . \tag{3.5}$$

We first integrate $\varphi_{M,H}$ to get $F_{M,H}$,

$$F_{M,H} = \int_{z_0}^{z} \frac{dz'}{z'} \varphi_{M,H}(z'/L)$$

$$= \frac{1}{2} \ln\left(\frac{z}{z_0}\right)$$

$$+ \sqrt{1+4\alpha\xi} - \sqrt{1+4\alpha\xi_0}$$

$$+ \frac{1}{2} \ln\left[\frac{(\sqrt{1+4\alpha\xi}-1)(\sqrt{1+4\alpha\xi_0}+1)}{(\sqrt{1+4\alpha\xi}+1)(\sqrt{1+4\alpha\xi_0}-1)}\right]. \quad (3.6)$$

Since $F_{M,H}$ are conventionally written in the form

$$F_{M,H} = \ln\frac{z}{z_0} - \Psi(z/L\,;\, z_0/L), \quad (3.7)$$

(Lumley and Panofsky, 1964), we see that $\frac{1}{2}\ln(z/z_0)$ has been 'lost' within the algebra of (3.6). Moreover, experience in computing the $n+1$ iteration of ξ ($\equiv \xi_{n+1}$) from

$$\xi_{n+1} = \frac{F_M^2(\xi_n\,;\,\xi_{0n})}{F_H(\xi_n\,;\,\xi_{0n})} R_B \quad (3.8)$$

shows that the iteration fails for close-to-neutral cases in which ξ_{0n} becomes too small. There is nothing algebraically incorrect with (2.7)

and (3.6); the difficulty arises from the computation of the factor $(\sqrt{1+4\alpha\xi_0}-1)$. When ξ_0 is sufficiently small, the factor is erroneously computed as a negative number with a very small absolute value on a computer with finite precision. This causes the bracketed factor in (3.6) to be a large negative quantity, and the computation halts.

To find a remedy for this failure, we must derive a more 'computer friendly' form of (3.6). This is achieved by writing the bracketed term in (3.6) as

$$\frac{1}{2}\ln\left[\frac{(s-1)(s_0+1)}{(s+1)(s_0-1)}\right]$$

where the s, s_0 notation is obvious, and by multiplying the numerator and denominator by $s_0 + 1$ and $s + 1$:

$$\frac{1}{2}\ln\left[\frac{(s^2-1)(s_0+1)^2}{(s+1)^2(4\alpha\xi_0)}\right] = \frac{1}{2}\ln\frac{Z}{Z_0} + \ln\left(\frac{s_0+1}{s+1}\right) \qquad (3.9)$$

We see that the 'missing' $\frac{1}{2}\ln(Z/Z_0)$ has been recovered, and we have the readily-computable (3.10):

$$F_{M,H} = \ln\left(\frac{Z}{Z_0}\right) - \Psi(Z/L;\, Z_0/L) \qquad (3.10)$$

with

$$\Psi = (1+4\alpha\xi_0)^{1/2} - (1+4\alpha\xi) + \ln\left(\frac{\sqrt{1+4\alpha\xi}+1}{\sqrt{1+4\alpha\xi_0}+1}\right). \tag{3.11}$$

$F_{M,H}$ reduces to the familiar 'log + linear' form as neutral conditions $(L \to +\infty)$ are approached:

$$F_{M,H} \longrightarrow \ln\frac{z}{z_0} + \alpha(z - z_0). \tag{3.12}$$

The solution for ξ for the log + linear expression is given by (2.12).

Eqs. (2.12) and (3.12) are only valid for $\alpha\xi \ll 1$. To derive an approximate expression valid for $\alpha\xi \gg 1$, we write $F_{M,H}$ as

$$\begin{aligned} F_{M,H}(\xi;\xi_0) &= \int_{\xi_0}^{\xi_b} \frac{d\xi'}{\xi'}\varphi_{M,H}(\xi') + \int_{\xi_b}^{\xi} \frac{d\xi'}{\xi'}\varphi_{M,H}(\xi') \\ &\equiv F_{M,H}(\xi_b;\xi_0) + Q(\xi,\xi_b) \end{aligned} \tag{3.13}$$

in which ξ_b is chosen so that $\alpha\xi_b \gg 1$. We then have

$$\begin{aligned} Q(\xi,\xi_b) &= \frac{1}{2}\int_{\xi_b}^{\xi} \frac{d\xi'}{\xi'}\left(1+\sqrt{1+4\alpha\xi}\right) \\ &\sim \frac{1}{2}\ln\frac{\xi}{\xi_b} + \sqrt{\alpha}\int_{\xi_b}^{\xi}\frac{d\xi'}{\xi'}\,\xi'^{\frac{1}{2}} \\ &= \frac{1}{2}\ln(z/z_b) + 2\sqrt{\alpha}\left(\xi^{1/2} - \xi_b^{1/2}\right). \end{aligned} \tag{3.14}$$

Thus, for $\alpha \xi_b >> 1$,

$$F_{M,H} \sim \ln\left(\frac{z_b}{z_o}\right) - \Psi(\xi_b; \xi_o) + \frac{1}{2}\ln\frac{z}{z_b} + 2\sqrt{\alpha}\left(\xi^{1/2} - \xi_b^{1/2}\right). \tag{3.15}$$

It is convenient and standard practice to define

$$\Psi(\xi, \xi_o) = \psi(\xi) - \psi(\xi_o) \tag{3.16}$$

in which

$$\psi(\xi) \equiv \lim_{z_o \to 0} \Psi(\xi; \xi_o). \tag{3.17}$$

For the problem at hand, $\psi(\xi)$ is given by

$$\psi(\xi) = \ln\left(1 + \sqrt{1 + 4\alpha\xi}\right) - (1 + 4\alpha\xi)^{1/2} + 1 - \ln 2. \tag{3.18}$$

The approximate expression for $F_{M,H}(\xi)$ can now be written as

$$F_{M,H}(\xi) \sim \ln\left(\frac{\xi_b}{\xi_o}\right) - \psi(\xi_b) + \psi(\xi_o)$$
$$+ \frac{1}{2}\ln\left(\frac{z}{z_b}\right) + 2\sqrt{\alpha}\left(\xi^{1/2} - \xi_b^{1/2}\right),$$

which, for $\alpha = 5$ and $\xi_b = 1$, becomes

$$F_{M,H}(\xi) \sim \ln\left(\frac{z}{z_0}\right) - \frac{1}{2}\ln\xi + 2\sqrt{5}\,\xi^{1/2} + \hat{A}, \tag{3.19}$$

$$\hat{A} \equiv \sqrt{21} - \ln(1+\sqrt{21}) - 2\sqrt{5} - 1 + \ln 2 \tag{3.20}$$

$$\doteq -1.916.$$

If $\frac{1}{2}\ln\xi$ is neglected in comparison to $\ln(z/z_0) + \hat{A}$, we get,

$$\xi \sim \left[\ln(z/z_0) + 2\sqrt{5}\,\xi^{1/2} + \hat{A}\right] R_B. \tag{3.21}$$

If we let $x \equiv \xi^{1/2}$, then (3.21) approximates the quadratic equation

$$x^2 - 2\sqrt{\alpha}\,R_B\,x - A R_B = 0 \tag{3.22}$$

$$A = \hat{A} + \ln(z/z_0). \tag{3.23}$$

The physically correct solution to (3.22) is

$$x_{(+)} = \sqrt{\alpha}\,R_B + \sqrt{\alpha R_B^2 + A R_B} \tag{3.24}$$

in which ξ_1 is defined as

$$\xi_1 \equiv \chi^2_{(+)} . \tag{3.25}$$

Numerical computations show that ξ can be adequately approximated by

$$\xi_\infty \equiv R_B \ln(z/z_0) \text{ for } \xi \leq 0.25 \text{ and } \xi_1 \text{ for } \xi > 0.25.$$

Substitution of the approximate value of ξ into (3.10,11) leads to momentum and heat transfer coefficients $C_{M,H}$ that differ only slightly from the 'exact' values. Table I shows the relation between

$$L, R_B, \xi, \xi_\infty (\equiv R_B \ln z/z_0), \xi_1 [\equiv \text{Eqs. } (3.24, 3.25)]$$

and $C_{M,H}$ for a wide range of L for Z = 50 m and for Z_0 = 0.0001 m. Table II is the same as Table I, except that Z_0 = 1.0 m.

These tables encompass the extreme ranges of stability and roughness that are likely to be encountered in routine modeling calculations, yet go well beyond the range for which there are supporting field data. For example, for $L \sim z_0 \sim 1$ meter, it is unlikely that any existing profile law can adequately predict θ (Z) and U (Z). In fact, all calculations for $z_0 \gtrsim 0.1\,|L|$ should be viewed with suspicion, even though such conditions may arise fairly frequently in numerical modeling practice.

Table I. Comparison between exact and approximate ξ for $z_0 = 0.0001$ m

L, meters	$R_B \times 10^3$	ξ	ξ_∞	ξ_1	$C_{M,H} \times 10^3$
10,000	0.38	5.00E-3	4.99E-3	4.37E-3	0.926
1,000	3.75	5.00E-2	4.92E-2	4.56E-2	0.898
500	7.38	10.00E-2	9.69E-2	9.28E-2	0.872
250	17.80	0.2	0.189	0.190	0.831
100	34.10	0.5	0.447	0.488	0.743
75	44.30	0.667	0.582	0.658	0.708
50	63.80	1.00	0.837	1.00	0.651
25	116.00	2.00	1.52	2.04	0.540
10	244.00	5.00	3.21	5.24	0.382
5	412.00	10.00	5.41	10.6	0.272
1	1220.00	50.00	16.0	53.7	0.0953

Table II. Comparison between exact and approximate ξ for $z_0 = 1.0$ m

L, meters	$R_B \times 10^3$	ξ	ξ_∞	ξ_1	$C_{M,H} \times 10^3$
10,000	1.27	5.00E-3	4.97E-3	2.84E-3	10.3
1,000	12.10	5.00E-2	4.73E-2	3.41E-3	9.37
500	23.10	10.00E-2	9.05E-2	7.44E-2	8.56
250	43.00	0.2	0.168	0.164	7.41
100	92.40	0.5	0.362	0.467	5.47
75	116.00	0.667	0.453	0.647	4.82
50	157.00	1.00	0.614	1.02	3.94
25	256.00	2.00	1.00	2.21	2.61
10	462.00	5.00	1.81	5.96	1.36
5	699.00	10.00	2.73	12.4	0.782
1	1710.00	50.00	6.70	65.3	0.188

Table III. Comparison of exact ($\mathcal{F}_H^e$) and approximate ($\mathcal{F}_H^a$) heat fluxes (watts m^{-2}) for $\Delta\Theta = 2$ K and $z_0 = 0.0001$ m

U	$R_B \times 10^3$	L	$C_{M,H} \times 10^3$	$\mathcal{F}_H^a$	$\mathcal{F}_H^e$
10	35.6	95.3	0.738	17.9	17.8
9	44.0	75.6	0.709	15.4	15.4
8	55.7	58.2	0.673	13.0	13.0
7	72.7	43.1	0.623	10.6	10.6
6	99.0	30.2	0.571	8.2	8.3
5	143.00	19.6	0.498	5.9	6.0
4	223.00	11.3	0.402	3.8	3.9
3	396.00	5.5	0.279	2.0	2.0
2	891.00	1.6	0.136	0.62	0.67
1	3560.00	0.2	0.021	0.05	0.05

Table IV. Similar to Table III except $z_0 = 1.0$ m

U	$R_B \times 10^3$	L	$C_{M,H} \times 10^3$	$\mathcal{F}_H^a$	$\mathcal{F}_H^e$
10	35.60	310.00	7.80	198.00	188.00
9	44.00	244.00	7.36	168.00	160.00
8	55.70	185.00	6.81	138.00	131.00
7	72.70	135.00	6.13	107.00	103.00
6	99.00	91.70	5.27	78.10	76.30
5	143.00	56.90	4.22	50.70	50.80
4	223.00	30.60	2.97	27.30	28.60
3	396.00	12.80	1.64	10.60	11.90
2	891.00	3.30	0.545	2.15	2.63
1	3560.00	0.20	0.501	0.09	0.12

Table I, representing a very smooth surface, shows that when L ranges from $1.0\,m$ (extremely stable) to $10^4\,m$ (very close to neutral), the heat and momentum transfer coefficients vary over about one order of magnitude. By constrast, Table II, representing an extremely rough surface, shows that the same range of L produces $C_{M,H}$ that span more than two orders of magnitude. We also see that the approximation $\xi \doteq \xi_\infty$ holds fairly well for $\xi \lesssim 0.25$, and that $\xi \sim \xi_1$ is reasonably valid for $\xi \gtrsim$ 0.25. Note that ξ_∞ progressively underestimates ξ_{exact} for $\xi_1 < 1$ and overestimates ξ_{exact} for $\xi_1 >$ 1.

Tables III and IV give the energy flux (watts m^{-2}) for $z_0 = 0.0001\,m$ and $z_0 = 1.0\,m$ for $\Delta\theta = 2\,K$ and U that diminishes from $10\,m\,sec^{-1}$ to $1\,m\,sec^{-1}$. Although the sensible heat flux for moderately stable conditions is rather modest, it is by no means negligible and is comparable to a typical nocturnal soil heat flux. In contrast, the sensible heat flux of 188 watts m^{-2} for $z_0 = 1.0\,m$ and $U \equiv 10\,m\,sec^{-1}$ is certainly counter-intuitive for nocturnal conditions. We note, however, that although R_B is stable, it is fairly close to neutral. In addition, a wind speed of $10\,m\,sec^{-1}$ over a very rough $z_0 = 1\,m$ that maintains a $\Delta\theta$ of 2 K is itself counter-intuitive. Counter-intuitive causes must be expected to produce counter-intuitive effects.

In general, the approximate expressions for ξ_∞ and ξ_1, produce fluxes that are very close to the 'exact' values.

IV Conclusions

A new method is presented for computing fluxes of momentum, heat, and moisture for the stable surface layer of the 'E2' physics of the MRF86

model. The new method achieves several goals: (1) The existing nocturnal 'E2' surface layer relations are incompatible with the formulation for turbulent transfer coefficients $(K_{M,H,Q})$ above the surface layer. The new method makes the relations compatible. (2) The current E2 physics subdivides the stable regime into weakly, mildly, and strongly stable subdomains, each with its own algebraic expressions. The new method unifies the stable regime into a single, continuous domain. (3) The E2 surface layer physics has a critical bulk Richardson number of 1.32, above which all fluxes vanish. The new method has no positive, finite critical bulk Richardson number. As the Richardson number increases, the surface fluxes decrease but do not vanish. (4) The E2 physics requires an iterative technique for calculating the surface fluxes. The new method does not.

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