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A Simpler Way to Initiate Condensation at Relative Humidities Below 100 Percent

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1. Background

Calculation of large-scale condensation in NMC models has typically used a saturation criteria set at a fraction of the saturation specific humidity, q_s:

$$q_s(\text{effective}) = \mu\, q_s(T,p) \tag{1}$$

For example, μ might have a value of 0.9. Forecast values of specific humidity q for a grid box are then compared with $\mu\, q_s(T,p)$ instead of q_s.

$$q \leq \mu\, q_s \quad : \text{ No saturation} \tag{2a}$$

$$q > q_s \quad : \text{ Saturation} \tag{2b}$$

This procedure presumably has been used to allow for the fact that atmospheric variables are not uniform in a grid box, that this is especially critical with respect to condensation, and that it is therefore unreasonable to delay the computed occurrence of saturation until $q=q_s$ in the entire box.

This procedure has several disadvantages, at least from a conceptual viewpoint.

a. Since q_s is largely proportional to the saturation vapor pressure $e_s(T)$, and the latter is largely proportional to the latent heat L, the factor μ has in effect reduced L. This could artifically reduce the destabilizing effect of large scale precipitation on stable large-scale motion by leading to smaller values of released latent heat.

b. The criteria (2) results in a relatively sudden onset of condensation throughout the grid box. This is not faithful to the (presumed) view of (1) as a device to allow for nonhomogeneity, in which condensation should first occur only in the moist parts of a grid box.

c. Since the large-scale q is limited by $\mu q_s < q_s$, the computation of a moist adiabat for use in moist convection parameterization, would, if based on a lifting-level computation, have to use (2) in determining the LCL. (If μ were not used in the LCL computation, the resulting moist adiabat would be colder than desirable). On the other hand, if the air in the layer that is lifted to its LCL already has a q almost equal to μq_s, there is no opportunity for (2) to lower the LCL as a compensation for the restriction $q \leq \mu q_s$. (i.e., the model has been prevented from obtaining q values between μq_s and q_s.)

Some of these problems can be "overcome" by ad-hoc devices. However, the original intent of (1) can be met by a very simple recognition of the variability of q in a grid box that seems to lead to fewer paradoxes requiring ad-hoc modifications than does the use of μ.

2. Treatment of Variable Specific Humidity

Let $\overline{q}$ represent the value of specific humidity

$$q = \frac{\epsilon e}{p - (1-\epsilon) e}, \tag{3}$$

$$\epsilon = R_d / R_v = 287.05/461.5$$

$$e = \text{vapor pressure}$$

that has been provisionally forecast for a grid box just prior to a condensation adjustment near the end of a time step (i.e. advection of q, small-scale turbulent mixing of q, and evaporation from the ground/ocean has been calculated). Let T represent the same type of value for temperature, including all effects except release of latent heat.

Before describing the treatment of a hypothesized non-uniform q, it is necessary to fix on the details of the condensation adjustment process for a <u>uniform</u> parcel. This is patterned after the familiar irreversible constant enthalpy isobaric process used for the thermodynamic treatment of the wet-bulb. Given the provisional T and the pressure p of the sample,

$$q_s(T) = \frac{\epsilon\, e_s(T)}{p - (1-\epsilon)\, e_s(T)} \tag{4}$$

can be computed. If $q_s > q$, no condensation occurs. If $q_s < q$ we expect condensation during which enthalpy is conserved and q is changed to

$$q_{new} = q_s(T_{new}) \tag{5}$$

From (4) we can derive the relation

$$\left(\frac{\partial q_s}{\partial T}\right)_p \approx \frac{L\, q_s}{R_v T^2} = \alpha\, q_s \tag{6}$$

The enthalpy conservation can then be stated

$$\begin{aligned} c_p(T_{new} - T) &= L\left[q - q_s(T_{new})\right] \\ &\approx L\left[q - q_s(T)\{1 + \alpha(T_{new} - T)\}\right]. \end{aligned} \tag{7}$$

This enables T_{new} and q_{new} to be computed

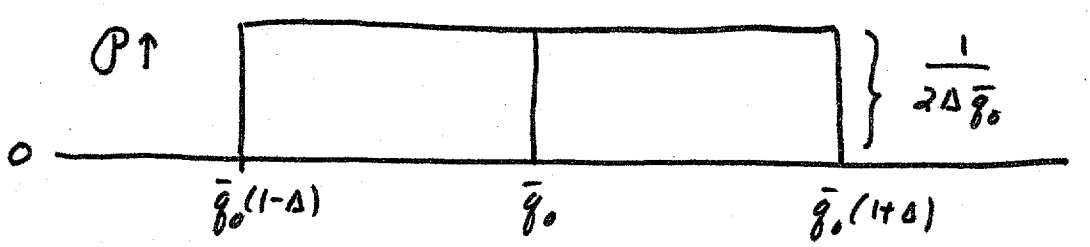
$$T_{new} = T + \beta \left[q - q_s(T) \right] \tag{8}$$

$$q_{new} = q - \frac{c_p}{L} (T_{new} - T) \tag{9}$$

where the factor β is given by

$$\beta = \frac{1}{\frac{c_p}{L} + \alpha q_s(T)} . \tag{10}$$

We now suppose that in a grid box the q values have an assumed distribution centered on the provisionally forecast value $\bar{q}_o$. For simplicity we specify this as a uniform distribution between $q_o(\min) = \bar{q}_o(1-\Delta)$ and $q_o(\max) = \bar{q}_o(1+\Delta)$, where Δ is an experimentally fixed constant.



$\mathcal{P}(q_1)$ is the probability that $q_1<q<q_1+dq$.

Given the uniform T and p of the grid box we can compute $q_s(T,p)$. Three cases exist.

<u>Case I</u>:

$$q_s > q_o(\max) = \bar{q}_o(1+\Delta) \tag{11}$$

No saturation occurs in this case.

Case II:

$$q_s < q_0(\min) = \bar{q}_0(1-\Delta) \tag{12}$$

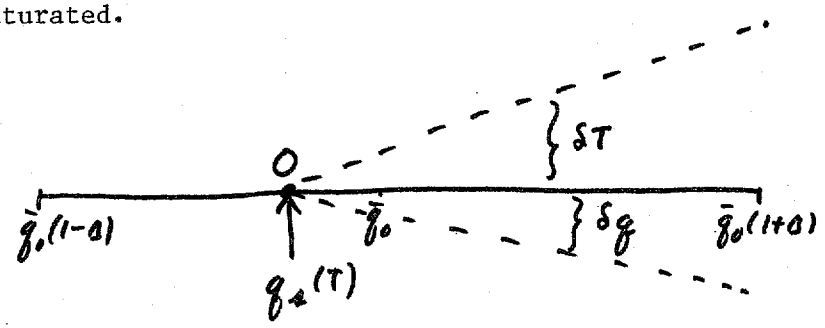
The entire box is saturated, so that (8) and (9) apply directly:

$$\begin{aligned} T_{new} &= T + \beta\left[\bar{q}_0 - q_s(T)\right], \\ \bar{q}_{new} &= \bar{q}_0 - \frac{c_p}{L}(T_{new} - T). \end{aligned} \tag{13}$$

Case III:

$$\bar{q}_0(1-\Delta) < q_s < \bar{q}_0(1+\Delta). \tag{14}$$

This corresponds to partial saturation of the box in which the portion with $q > q_s$ is saturated.



(15)

In the above diagram the saturated point to the right of point 0 will occupy a fraction

$$f = \frac{\bar{q}_0(1+\Delta) - q_s}{2\Delta\bar{q}_0} \tag{16}$$

of the box. We imagine that firstly, each saturated element to the right of 0 will experience a temperature change δT and moisture change δq given by (8) and (9) for its initial q and T:

$$\delta T = \beta\left[q - q_s(T)\right], \quad \delta q = -\frac{c_p}{L}\delta T. \tag{17}$$

would be the same as if Δ were zero. In a weak process we expect to find q_s between $\bar{q}(1-\Delta)$ and $\bar{q}(1+\Delta)$.

The simplest example to model is a hypothetical case of "pseudo-adiabatic" expansion. The change in pressure δp experienced by a parcel in a time step δt can be related to convergence fields in a numerical model by setting

$$\delta p = \omega \delta t \sim \Delta p \left| \frac{\partial \omega}{\partial p} \right| \delta t$$

$$\sim \Delta p \, |\nabla \cdot \vec{v}| \, \frac{\Delta x}{\sqrt{2}\, c}$$

where Δp=50 obs, c =300msec is the speed of an external gravity wave, and Δx is the horizontal grid increment of the model. For c=300 $msec^{-1}$ and Δx=100km, $\delta p(cb) = .12 \times 10^5 |\nabla \cdot \vec{v}| (sec^{-1})$ and δt = 4 minutes. We can therefore consider a pressure decrease of 1 cb per time step as arising from a very intense low level convergence field of size $0.8 \times 10^{-4} sec^{-1}$, while a change of 0.1 cb per time step corresponds to a horizontal convergence of $0.8 \times 10^{-5} sec^{-1}$. The former approaches the expansion rates in the eye wall of a hurricane, and the latter is like that expected in an intense extratropical cyclone.

Table I shows the results of carrying out the expansion with δp = 1 cb starting with a parcel at 100 cb, T=25°C, and a specific humidity of 0.017921. The initial relative humidity is 90 per cent, and the condensation begins at 97.4 cbs.* The first three columns show the adjusted T (Centigrade) and

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* Saturation vapor pressures were computed with the formula

$$e_s(cb) = 0.611 \left(T_3/T \right)^a \exp\left[(a+b)\left(1 - \frac{T_3}{T}\right) \right],$$

where

T_3 = 273.16 (triple point)
a = 5.0065 = $(C-C_{pv})/R_v$
b=19.83923 = $L_{v3}/R_v T_3$.

c = 4187 kj ton^{-1} deg^{-1} is the specific heat for liquid water and L_{v3} = 2.501×10^6 kj ton^{-1} is the latent heat of vaporization at the triple point. Dry adiabatic expansions were computed with the customary approximation of using the R/C_p ratio for dry air.

Secondly, we imagine that these individual changes are averaged over the box to produce the net changes in T and $\bar{q}$. Thus

$$
\begin{aligned}
T_{new} &= T + f \cdot \delta T_{mean} \\
&= T + f \cdot \tfrac{1}{2} \delta T_{max} \\
&= T + \tfrac{f}{2} \beta \left[(1+\Delta)\bar{q}_0 - q_s \right] \\
&= T + \beta \bar{q}_0 \Delta f^2 , \qquad (18)
\end{aligned}
$$

$$
\bar{q}_{new} = \bar{q}_0 - \frac{c_p}{L}(T_{new} - T) . \qquad (19)
$$

(Note that (18) agrees with (13) when f=1 at $q_s = \bar{q}_0(1-\Delta)$.)

The virtues of this approach over the saturation criterion device are primarily conceptual in nature.

a. The number Δ can logically be set proportional to the size $\Delta x \Delta y \Delta p$ of the grid element.

b. The conflict between (1) and moist adiabatic lapse rates for convection calculations is readily resolved by the simple device of assuming that q(max) = $\bar{q}$ (1+Δ) should be used to get the LCL for a cloud parcel, on the natural assumption that it is the moistest regions in a box that are likely to become cumulus clouds.

c. The f^2 term in (18) means that condensation will begin as a very gradual process.

3. Examples

The behavior of this condensation scheme depends on the location of q_s with respect to the assumed range $\bar{q}(1-\Delta)$ to $\bar{q}(1+\Delta)$ at the time of a saturation adjustment. This location will in turn depend on the intensity of the meteorological process causing condensation. In a very intense process we must expect that q_s will tend to be smaller than $\bar{q}(1-\Delta)$, and the new adjustment calculation

adjusted $\bar{q}$ as computed for $\Delta = 0$, i.e., as a straightforward "pseudo-adiabatic" computation. (These results are indistinguishable from the lines on the WB1041 thermodynamic chart used at NMC.) The next 8 columns show the differences ($dT=T(\Delta)-T(0)$ and $d\bar{q} = \bar{q}(\Delta)-\bar{q}(0)$) of computations made with the indicated value of Δ from those with $\Delta = 0$. The differences are small. Typically dT reaches a maximum (because $\Delta > 0$ implies earlier attainment of saturation than when $\Delta = 0$), and then decreases. The f values quickly became equal to 1 for $\Delta = .025$ but only slowly reached a value of 0.94 at 50 cbs for $\Delta = 0.1$ (The larger Δ values evidently define some fraction of $\bar{q}$ as unsaturated even for this large δp.)

Table II shows the same results when $\delta p=0.1$ cb (Only every 10th result is listed for easy comparison with Table I.)** The dT and $d\bar{q}$ values are larger than in Table I. Maximum f values were attained at 50 cb (0.61, 0.44, 0.36 and 0.31) but were significantly smaller than for $\delta p=1$ cb. This is because the smaller δp values correspond to a smaller dry adiabatic decrease of T and a smaller decrease in q_s in a single time step.

For the very intense adiabatic lifting process listed in Table I the process curves for $0<\Delta\leq 0.1$ are only slightly warmer than the "true" moist adiabatic give by $\Delta=0$. In the moderately intense process listed in Table II, the process curves for $0<\Delta\leq 0.1$ are warmer by up to 0.7°C than the "true" moist adiabat in the lower levels.

** There is a slight difference between the T and $\bar{q}$ for $\Delta=0$ on this table from those in Table I, equal , at 50 cb, to about 10% of the changes that occur in $\delta p=1$ cb. This can be viewed as a "truncation error" arising from the finite size of δt or δp. The difference of 0.9° in the second column at 50 cb suggests that some of the negative dT's in Table I are caused by this "truncation error".

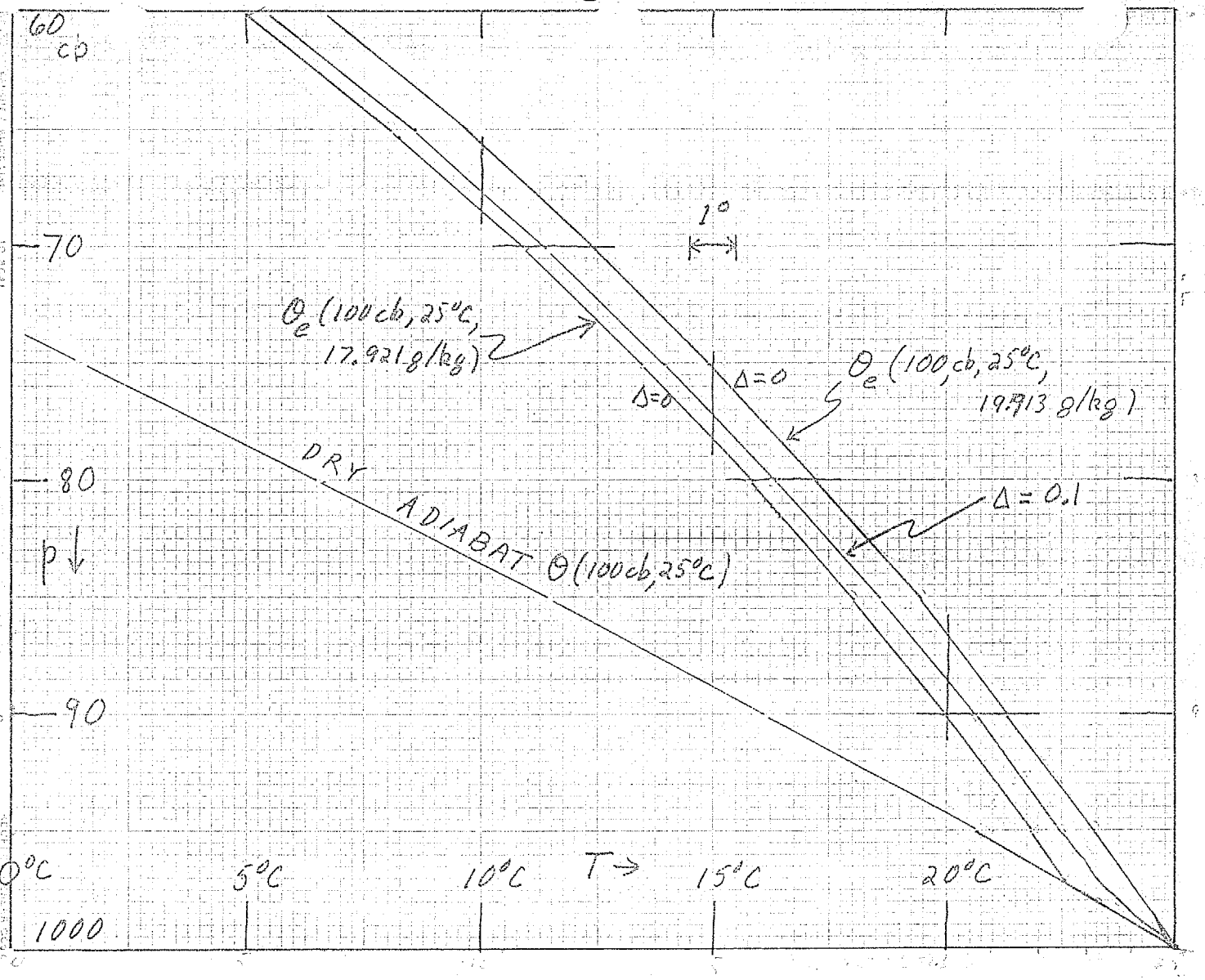

Fig. 1 Process curves for samples undergoing successive adiabatic pressure drops of 0.1 cb. The two outer curves are for $\Delta = 0$ computations starting with 90% and 100% relative humidity. The intermediate curve is for $\Delta = 0.1$ starting with $\bar{q}_0 = 90\%$ of q_s. A dry adiabat is shown for reference. The warmest $\Delta = 0$ curve is suggested as the appropriate "cloud" curve for Kuo convection parameterization.

Table I. $\delta p = 1$ cb.

	$\Delta = 0.0$		$\Delta = 0.025$		$\Delta = 0.05$		$\Delta = 0.075$		$\Delta = 0.1$	
p	T(°C)	$\bar{q}$	dT	$d\bar{q}$	dT	$d\bar{q}$	dT	$d\bar{q}$	dT	$d\bar{q}$
99	24.145264	0.017921	0.0	0.0	0.0	0.0	0.002930	-0.000001	0.032959	-0.000013
98	23.254180	0.017921	0.000977	-0.000000	0.045898	-0.000018	0.111328	-0.000045	0.197998	-0.000080
97	22.666504	0.017821	0.002197	-0.000001	0.065186	-0.000026	0.159668	-0.000065	0.271240	-0.000109
96	22.307861	0.017614	0.0	-0.000000	0.009277	-0.000004	0.081299	-0.000033	0.187256	-0.000076
95	21.942871	0.017405	0.0	-0.000000	0.003906	-0.000002	0.059326	-0.000024	0.155029	-0.000063
94	21.573486	0.017196	0.0	-0.000000	0.002930	-0.000001	0.052490	-0.000022	0.141602	-0.000058
93	21.199463	0.016985	0.0	-0.000000	0.002441	-0.000001	0.049072	-0.000020	0.134521	-0.000055
92	20.820557	0.016772	0.0	-0.000000	0.001953	-0.000001	0.046631	-0.000019	0.129639	-0.000053
91	20.436768	0.016558	0.0	-0.000000	0.001709	-0.000001	0.044434	-0.000019	0.125488	-0.000052
90	20.048096	0.016342	0.0	-0.000000	0.001221	-0.000000	0.042236	-0.000018	0.121338	-0.000050
89	19.654053	0.016125	0.0	0.0	0.000977	-0.000000	0.040283	-0.000017	0.117432	-0.000049
88	19.254883	0.015906	0.0	0.0	0.000732	-0.000000	0.038086	-0.000016	0.113525	-0.000048
87	18.850098	0.015686	0.0	0.0	0.000488	-0.000000	0.036133	-0.000015	0.109619	-0.000046
86	18.439697	0.015464	0.0	0.0	0.000244	-0.000000	0.033936	-0.000015	0.105713	-0.000045
85	18.023438	0.015240	0.0	0.0	0.000244	-0.000000	0.031982	-0.000014	0.102051	-0.000043
84	17.601318	0.015014	0.0	0.0	0.0	0.000000	0.030029	-0.000013	0.098145	-0.000042
83	17.173096	0.014787	0.0	0.0	0.0	0.000000	0.027832	-0.000012	0.094238	-0.000041
82	16.738281	0.014559	0.0	0.0	0.0	0.000000	0.026123	-0.000012	0.090576	-0.000039
81	16.297119	0.014328	0.0	0.0	0.0	0.000000	0.024170	-0.000011	0.086670	-0.000038
80	15.849121	0.014096	0.0	0.0	0.0	0.000000	0.022217	-0.000010	0.082764	-0.000036
79	15.394043	0.013862	0.0	0.0	0.0	0.000000	0.020508	-0.000010	0.079102	-0.000035
78	14.931885	0.013627	0.0	0.0	0.0	0.000000	0.018799	-0.000009	0.075439	-0.000034
77	14.462158	0.013389	0.0	0.0	0.0	0.000000	0.017090	-0.000008	0.071777	-0.000032
76	13.984619	0.013150	0.0	0.0	0.0	0.0	0.015625	-0.000008	0.068115	-0.000031
75	13.499268	0.012910	0.0	0.0	0.0	0.0	0.013916	-0.000007	0.064453	-0.000030
74	13.005615	0.012667	0.0	0.0	0.0	0.0	0.012207	-0.000007	0.060791	-0.000028
73	12.503174	0.012423	0.0	0.0	0.0	0.0	0.010742	-0.000006	0.057373	-0.000027
72	11.991943	0.012177	0.0	0.0	0.0	0.0	0.009521	-0.000005	0.053955	-0.000026
71	11.471436	0.011929	0.0	0.0	0.0	0.0	0.008301	-0.000005	0.050537	-0.000024
70	10.941406	0.011679	0.0	0.0	0.0	0.0	0.007080	-0.000004	0.047119	-0.000023
69	10.401611	0.011428	0.0	0.0	0.0	0.0	0.005615	-0.000004	0.043457	-0.000022
68	9.851318	0.011174	0.0	0.0	0.0	0.0	0.004395	-0.000003	0.040283	-0.000020
67	9.290283	0.010920	0.0	0.0	0.0	0.0	0.003418	-0.000003	0.036865	-0.000019
66	8.717773	0.010663	0.0	0.0	0.0	0.0	0.002686	-0.000003	0.033936	-0.000018
65	8.133789	0.010404	0.0	0.0	0.0	0.0	0.001709	-0.000002	0.030762	-0.000017
64	7.537598	0.010144	0.0	0.0	0.0	0.0	0.000977	-0.000002	0.027832	-0.000016
63	6.928711	0.009883	0.0	0.0	0.0	0.0	0.000488	-0.000002	0.024902	-0.000015
62	6.306641	0.009619	0.0	0.0	0.0	0.0	-0.000244	-0.000001	0.021973	-0.000014
61	5.670654	0.009354	0.0	0.0	0.0	0.0	-0.000732	-0.000001	0.019043	-0.000012
60	5.020020	0.009088	0.0	0.0	0.0	0.0	-0.000977	-0.000001	0.016602	-0.000011
59	4.354248	0.008820	0.0	0.0	0.0	0.0	-0.001221	-0.000001	0.014160	-0.000010
58	3.672607	0.008550	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.011719	-0.000009
57	2.974121	0.008280	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.009521	-0.000008
56	2.258301	0.008008	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.007080	-0.000008
55	1.523926	0.007734	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.005127	-0.000007
54	0.770020	0.007460	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.003174	-0.000006
53	-0.004150	0.007185	0.0	0.0	0.0	0.0	-0.001709	-0.000001	0.001221	-0.000005
52	-0.800049	0.006909	0.0	0.0	0.0	0.0	-0.001465	-0.000001	-0.000244	-0.000005
51	-1.618652	0.006632	0.0	0.0	0.0	0.0	-0.001465	-0.000001	-0.001709	-0.000004
50	-2.461182	0.006355	0.0	0.0	0.0	0.0	-0.001709	-0.000001	-0.002930	-0.000004

The pseudo-moist adiabat (Δ=0.) obtained by using δp=0.1 cb but starting with p=100, T=25°C and a moister $\overline{q}_o$ corresponding to (1.1)x (.017921) represents the process undergone by a saturated parcel whose moisture is equal to the moistest extreme of the Δ= 0.1 set of Table II. Its temperatures are warmer than those for the $\overline{q}_o$ = 0.17921 pseudo-moist adiabat given in column 2 of Table II by an amount equal to 1.26° at 95 cb and 1.99° at 50 cb. Therefore the temperature differences shown in the next to last column of Table II indicate that the process curve for Δ =0.1 lies between the pseudo moist adiabatic having an initial $\overline{q}_o$=.017921 and that with $\overline{q}_o$=(1.1)X(.017921), as one might expect. Therefore, if one uses the moist extreme $q_{max} = (1+\Delta)\overline{q}$ to compute the moist adiabat for a convective parameterization process, it will give a T profile that is slightly warmer than the large-scale process curve computed with Δ>0. This is desirable because one does not want the large scale motion to have a more unstable (warmer) process curve than that assumed to exist for an individual cloud. The differences are shown on Fig. 1.

The above examples of rapid decrease of pressure on a moist sample have been computed to show that (for at least $\Delta \leq 0.1$) this description of the saturation process does not produce large scale process curves that are significantly different from a moist adiabat. This knowledge is necessary in order that parameterization of moist convection - a step taken to eliminate ordinary moist instability on the large scale - will not have to cope with any unusual instability properties that are introduced by the use of Δ . A more common condensation occurrence is that of slow ascent of stable saturated air in mid troposphere in a typical extratropical cyclone. This can be modelled simply by a weaker expansion with δp = 0.02 cb, corresponding to an upward vertical velocity of 1cm/sec at 70 cb and a time step of 4 minutes. The

Table II. $\delta p = 0.1\,cb$

	$\Delta = 0.0$		$\Delta = 0.025$		$\Delta = 0.05$		$\Delta = 0.075$		$\Delta = 0.1$	
P	T(°C)	$\bar{g}$	dT	$d\bar{g}$	dT	$d\bar{g}$	dT	$d\bar{g}$	dT	$d\bar{g}$
99	24.143799	0.017921	0.0	0.0	0.0	0.0	0.003662	-0.000002	0.080811	-0.000033
98	23.281494	0.017921	0.000977	-0.000000	0.093018	-0.000038	0.271484	-0.000110	0.477539	-0.000193
97	22.665283	0.017820	0.102295	-0.000041	0.293213	-0.000119	0.502686	-0.000203	0.718994	-0.000291
96	22.303955	0.017612	0.099121	-0.000041	0.288086	-0.000117	0.496338	-0.000202	0.711426	-0.000289
95	21.937988	0.017403	0.097656	-0.000040	0.285156	-0.000116	0.491455	-0.000200	0.704590	-0.000287
94	21.567627	0.017193	0.096191	-0.000040	0.281982	-0.000115	0.486572	-0.000199	0.697998	-0.000286
93	21.192383	0.016981	0.094727	-0.000039	0.278809	-0.000114	0.481689	-0.000197	0.691406	-0.000284
92	20.812500	0.016767	0.093262	-0.000039	0.275391	-0.000113	0.476563	-0.000196	0.684570	-0.000282
91	20.427734	0.016552	0.091797	-0.000038	0.272217	-0.000112	0.471680	-0.000194	0.677490	-0.000280
90	20.037842	0.016336	0.090332	-0.000038	0.269043	-0.000111	0.466309	-0.000193	0.671143	-0.000278
89	19.642822	0.016118	0.088867	-0.000037	0.265381	-0.000110	0.461182	-0.000191	0.663818	-0.000276
88	19.242432	0.015898	0.087402	-0.000037	0.262207	-0.000109	0.455811	-0.000190	0.656982	-0.000274
87	18.836670	0.015677	0.085693	-0.000036	0.258545	-0.000108	0.450439	-0.000188	0.649658	-0.000272
86	18.425049	0.015454	0.084229	-0.000036	0.255127	-0.000107	0.445068	-0.000187	0.642578	-0.000270
85	18.007568	0.015229	0.082764	-0.000035	0.251709	-0.000106	0.439697	-0.000185	0.635254	-0.000267
84	17.584229	0.015003	0.081055	-0.000035	0.247803	-0.000105	0.434326	-0.000183	0.627686	-0.000265
83	17.154541	0.014775	0.079590	-0.000034	0.244385	-0.000104	0.428711	-0.000182	0.620361	-0.000263
82	16.718506	0.014546	0.078125	-0.000033	0.240967	-0.000103	0.423096	-0.000180	0.612793	-0.000261
81	16.275879	0.014315	0.076660	-0.000033	0.237305	-0.000101	0.417725	-0.000178	0.604980	-0.000258
80	15.826416	0.014082	0.075195	-0.000032	0.233643	-0.000100	0.412109	-0.000176	0.597412	-0.000256
79	15.370117	0.013847	0.073486	-0.000031	0.229980	-0.000099	0.406006	-0.000174	0.589355	-0.000253
78	14.906494	0.013611	0.071777	-0.000031	0.226074	-0.000098	0.400146	-0.000172	0.581299	-0.000251
77	14.435303	0.013373	0.070313	-0.000030	0.222168	-0.000097	0.394287	-0.000171	0.572998	-0.000249
76	13.956299	0.013133	0.068848	-0.000030	0.218262	-0.000095	0.388184	-0.000168	0.564697	-0.000246
75	13.469238	0.012891	0.067627	-0.000029	0.214355	-0.000094	0.382324	-0.000166	0.556396	-0.000244
74	12.973877	0.012648	0.065918	-0.000028	0.210693	-0.000093	0.375977	-0.000164	0.547852	-0.000241
73	12.469971	0.012403	0.064209	-0.000027	0.206787	-0.000091	0.369629	-0.000162	0.539063	-0.000238
72	11.957031	0.012156	0.062744	-0.000027	0.202637	-0.000090	0.363525	-0.000160	0.530518	-0.000235
71	11.434814	0.011907	0.061279	-0.000026	0.198975	-0.000088	0.356934	-0.000158	0.521729	-0.000232
70	10.903076	0.011657	0.059326	-0.000026	0.194824	-0.000087	0.350342	-0.000156	0.512207	-0.000230
69	10.361328	0.011405	0.057617	-0.000025	0.190674	-0.000085	0.343506	-0.000154	0.502930	-0.000227
68	9.809082	0.011151	0.055908	-0.000024	0.186279	-0.000084	0.336670	-0.000151	0.493896	-0.000224
67	9.246094	0.010895	0.054199	-0.000024	0.182129	-0.000082	0.329590	-0.000149	0.484131	-0.000221
66	8.672119	0.010638	0.052246	-0.000023	0.177734	-0.000081	0.322510	-0.000147	0.474121	-0.000218
65	8.086182	0.010379	0.050293	-0.000023	0.173096	-0.000079	0.315186	-0.000145	0.464111	-0.000215
64	7.488037	0.010119	0.048584	-0.000022	0.168701	-0.000078	0.307617	-0.000142	0.453857	-0.000211
63	6.876953	0.009856	0.047119	-0.000021	0.164063	-0.000076	0.300537	-0.000140	0.443848	-0.000208
62	6.252441	0.009592	0.045410	-0.000020	0.159912	-0.000075	0.293213	-0.000137	0.433594	-0.000204
61	5.614258	0.009327	0.043701	-0.000020	0.155273	-0.000073	0.285400	-0.000135	0.422852	-0.000201
60	4.961182	0.009060	0.041992	-0.000019	0.150635	-0.000071	0.277832	-0.000132	0.412354	-0.000197
59	4.293213	0.008791	0.040039	-0.000018	0.145508	-0.000070	0.269775	-0.000130	0.401367	-0.000194
58	3.609131	0.008522	0.038086	-0.000018	0.140381	-0.000068	0.261719	-0.000127	0.389893	-0.000190
57	2.908203	0.008250	0.036377	-0.000017	0.135742	-0.000066	0.253418	-0.000124	0.378418	-0.000186
56	2.189697	0.007978	0.034912	-0.000016	0.130615	-0.000065	0.245117	-0.000121	0.366943	-0.000182
55	1.452393	0.007704	0.033447	-0.000016	0.125977	-0.000063	0.236816	-0.000118	0.355225	-0.000178
54	0.695801	0.007430	0.031494	-0.000015	0.120850	-0.000061	0.228271	-0.000115	0.343018	-0.000174
53	-0.081543	0.007154	0.029785	-0.000014	0.115967	-0.000059	0.219482	-0.000112	0.330322	-0.000170
52	-0.880859	0.006878	0.028076	-0.000013	0.111084	-0.000057	0.211426	-0.000109	0.318604	-0.000165
51	-1.703125	0.006601	0.026367	-0.000013	0.106445	-0.000055	0.202881	-0.000106	0.307129	-0.000160
50	-2.549072	0.006324	0.024414	-0.000012	0.101563	-0.000053	0.193848	-0.000102	0.293457	-0.000155

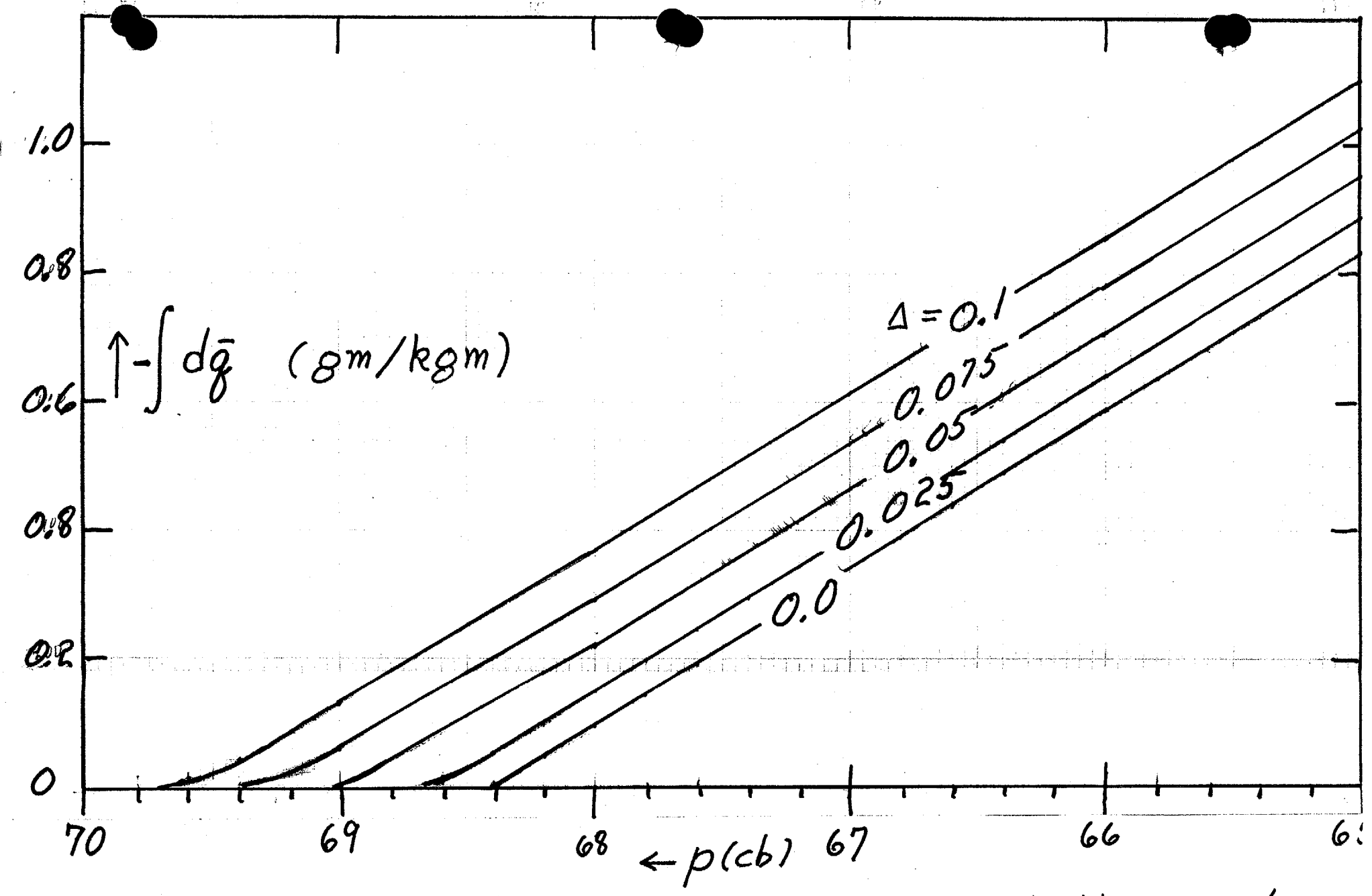

Fig. 2. Accumulated condensate for 5 values of Δ experienced by a sample starting at 70 cb with $T = 10°C$ and a relative humidity of 90%. ($\bar{q} = .009873$) when subjected to successive adiabatic pressure drops of 0.02 cb.

interest here is in the <u>time of onset</u> of precipitation. I took a sample at 70 cb, T = 10°C and a specific humidity of 9.873 gm/kgm (a relative humidity of 90 per cent). The attached figure shows the accumulated condensate ($9.873-\overline{q}$) as a function of pressure for the 5 Δ-values over the 16 hour period it would take this sample to be lifted to 65 cb.

The five curves are very simple. If we ignore the short initial curved part on the $\Delta=0.1$ curve, its straight line extension would intersect the x-axis at p=69.55 cb. The $\Delta=0$ expansion of this sample reached saturation at 68.4 cb. The parallelism of the $\Delta=0.1$ and $\Delta=0.0$ curves suggests that we may consider the $\Delta=0.1$ curve to be partly equivalent to the use of a reduced saturation criterion μ, as discussed at the beginning of this note. We can then rewrite the approximate lifting condensation level ratios between the $\Delta=0$ curve and the $\Delta=0.1$ curve in terms of μq_s:

$$\frac{q_s - q}{70-68.4} = \frac{\mu q_s - q}{70-69.55}$$

(The pressure decrease necessary for saturation is proportional to the saturation deficit.) Replacing q by 0.9 q_s we find

$$\mu\,(\Delta \overset{\text{for}}{=} 0.1) = 0.9 + \frac{.45 \times .1}{1.6} = 0.93$$

This relation is comfortably simple but it must be remembered that the major differences between using the saturation criterion μ and the Δ concept is that the former lowers q_s (and then $\overline{q}$) while the latter acknowledges the presence of q values in excess of $\overline{q}$ and does not change the fundamental definition of q_s.

Table II. $\delta p = 0.1\,cb$

	$\Delta = 0.0$		$\Delta = 0.025$		$\Delta = 0.05$		$\Delta = 0.075$		$\Delta = 0.1$	
P	T(°C)	$\bar{q}$	dT	$d\bar{q}$	dT	$d\bar{q}$	dT	$d\bar{q}$	dT	$d\bar{q}$
99	24.143799	0.017921	0.0	0.0	0.0	0.0	0.003662	-0.000002	0.080811	-0.000033
98	23.281494	0.017921	0.000977	-0.000000	0.093018	-0.000038	0.271484	-0.000110	0.477539	-0.000193
97	22.665283	0.017820	0.102295	-0.000041	0.293213	-0.000119	0.502686	-0.000203	0.718994	-0.000291
96	22.303955	0.017612	0.099121	-0.000041	0.288086	-0.000117	0.496338	-0.000202	0.711426	-0.000289
95	21.937988	0.017403	0.097656	-0.000040	0.285156	-0.000116	0.491455	-0.000200	0.704590	-0.000287
94	21.567627	0.017193	0.096191	-0.000040	0.281982	-0.000115	0.486572	-0.000199	0.697998	-0.000286
93	21.192383	0.016981	0.094727	-0.000039	0.278809	-0.000114	0.481689	-0.000197	0.691406	-0.000284
92	20.812500	0.016767	0.093262	-0.000039	0.275391	-0.000113	0.476563	-0.000196	0.684570	-0.000282
91	20.427734	0.016552	0.091797	-0.000038	0.272217	-0.000112	0.471680	-0.000194	0.677490	-0.000280
90	20.037842	0.016336	0.090332	-0.000038	0.269043	-0.000111	0.466309	-0.000193	0.671143	-0.000278
89	19.642822	0.016118	0.088867	-0.000037	0.265381	-0.000110	0.461182	-0.000191	0.663818	-0.000276
88	19.242432	0.015898	0.087402	-0.000037	0.262207	-0.000109	0.455811	-0.000190	0.656982	-0.000274
87	18.836670	0.015677	0.085693	-0.000036	0.258545	-0.000108	0.450439	-0.000188	0.649658	-0.000272
86	18.425049	0.015454	0.084229	-0.000036	0.255127	-0.000107	0.445068	-0.000187	0.642578	-0.000270
85	18.007568	0.015229	0.082764	-0.000035	0.251709	-0.000106	0.439697	-0.000185	0.635254	-0.000267
84	17.584229	0.015003	0.081055	-0.000035	0.247803	-0.000105	0.434326	-0.000183	0.627686	-0.000265
83	17.154541	0.014775	0.079590	-0.000034	0.244385	-0.000104	0.428711	-0.000182	0.620361	-0.000263
82	16.718506	0.014546	0.078125	-0.000033	0.240967	-0.000103	0.423096	-0.000180	0.612793	-0.000261
81	16.275879	0.014315	0.076660	-0.000033	0.237305	-0.000101	0.417725	-0.000178	0.604980	-0.000258
80	15.826416	0.014082	0.075195	-0.000032	0.233643	-0.000100	0.412109	-0.000176	0.597412	-0.000256
79	15.370117	0.013847	0.073486	-0.000031	0.229980	-0.000099	0.406006	-0.000174	0.589355	-0.000253
78	14.906494	0.013611	0.071777	-0.000031	0.226074	-0.000098	0.400146	-0.000172	0.581299	-0.000251
77	14.435303	0.013373	0.070313	-0.000030	0.222168	-0.000097	0.394287	-0.000171	0.572998	-0.000249
76	13.956299	0.013133	0.068848	-0.000030	0.218262	-0.000095	0.388184	-0.000168	0.564697	-0.000246
75	13.469238	0.012891	0.067627	-0.000029	0.214355	-0.000094	0.382324	-0.000166	0.556396	-0.000244
74	12.973877	0.012648	0.065918	-0.000028	0.210693	-0.000093	0.375977	-0.000164	0.547852	-0.000241
73	12.469971	0.012403	0.064209	-0.000027	0.206787	-0.000091	0.369629	-0.000162	0.539063	-0.000238
72	11.957031	0.012156	0.062744	-0.000027	0.202637	-0.000090	0.363525	-0.000160	0.530518	-0.000235
71	11.434814	0.011907	0.061279	-0.000026	0.198975	-0.000088	0.356934	-0.000158	0.521729	-0.000232
70	10.903076	0.011657	0.059326	-0.000026	0.194824	-0.000087	0.350342	-0.000156	0.512207	-0.000230
69	10.361328	0.011405	0.057617	-0.000025	0.190674	-0.000085	0.343506	-0.000154	0.502930	-0.000227
68	9.809082	0.011151	0.055908	-0.000024	0.186279	-0.000084	0.336670	-0.000151	0.493896	-0.000224
67	9.246094	0.010895	0.054199	-0.000024	0.182129	-0.000082	0.329590	-0.000149	0.484131	-0.000221
66	8.672119	0.010638	0.052246	-0.000023	0.177734	-0.000081	0.322510	-0.000147	0.474121	-0.000218
65	8.086182	0.010379	0.050293	-0.000023	0.173096	-0.000079	0.315186	-0.000145	0.464111	-0.000215
64	7.488037	0.010119	0.048584	-0.000022	0.168701	-0.000078	0.307617	-0.000142	0.453857	-0.000211
63	6.876953	0.009856	0.047119	-0.000021	0.164063	-0.000076	0.300537	-0.000140	0.443848	-0.000208
62	6.252441	0.009592	0.045410	-0.000020	0.159912	-0.000075	0.293213	-0.000137	0.433594	-0.000204
61	5.614258	0.009327	0.043701	-0.000020	0.155273	-0.000073	0.285400	-0.000135	0.422852	-0.000201
60	4.961182	0.009060	0.041992	-0.000019	0.150635	-0.000071	0.277832	-0.000132	0.412354	-0.000197
59	4.293213	0.008791	0.040039	-0.000018	0.145508	-0.000070	0.269775	-0.000130	0.401367	-0.000194
58	3.609131	0.008522	0.038086	-0.000018	0.140381	-0.000068	0.261719	-0.000127	0.389893	-0.000190
57	2.908203	0.008250	0.036377	-0.000017	0.135742	-0.000066	0.253418	-0.000124	0.378418	-0.000186
56	2.189697	0.007978	0.034912	-0.000016	0.130615	-0.000065	0.245117	-0.000121	0.366943	-0.000182
55	1.452393	0.007704	0.033447	-0.000016	0.125977	-0.000063	0.236816	-0.000118	0.355225	-0.000178
54	0.695801	0.007430	0.031494	-0.000015	0.120850	-0.000061	0.228271	-0.000115	0.343018	-0.000174
53	-0.081543	0.007154	0.029785	-0.000014	0.115967	-0.000059	0.219482	-0.000112	0.330322	-0.000170
52	-0.880859	0.006878	0.028076	-0.000013	0.111084	-0.000057	0.211426	-0.000109	0.318604	-0.000165
51	-1.703125	0.006601	0.026367	-0.000013	0.106445	-0.000055	0.202881	-0.000106	0.307129	-0.000160
50	-2.549072	0.006324	0.024414	-0.000012	0.101563	-0.000053	0.193848	-0.000102	0.293457	-0.000155

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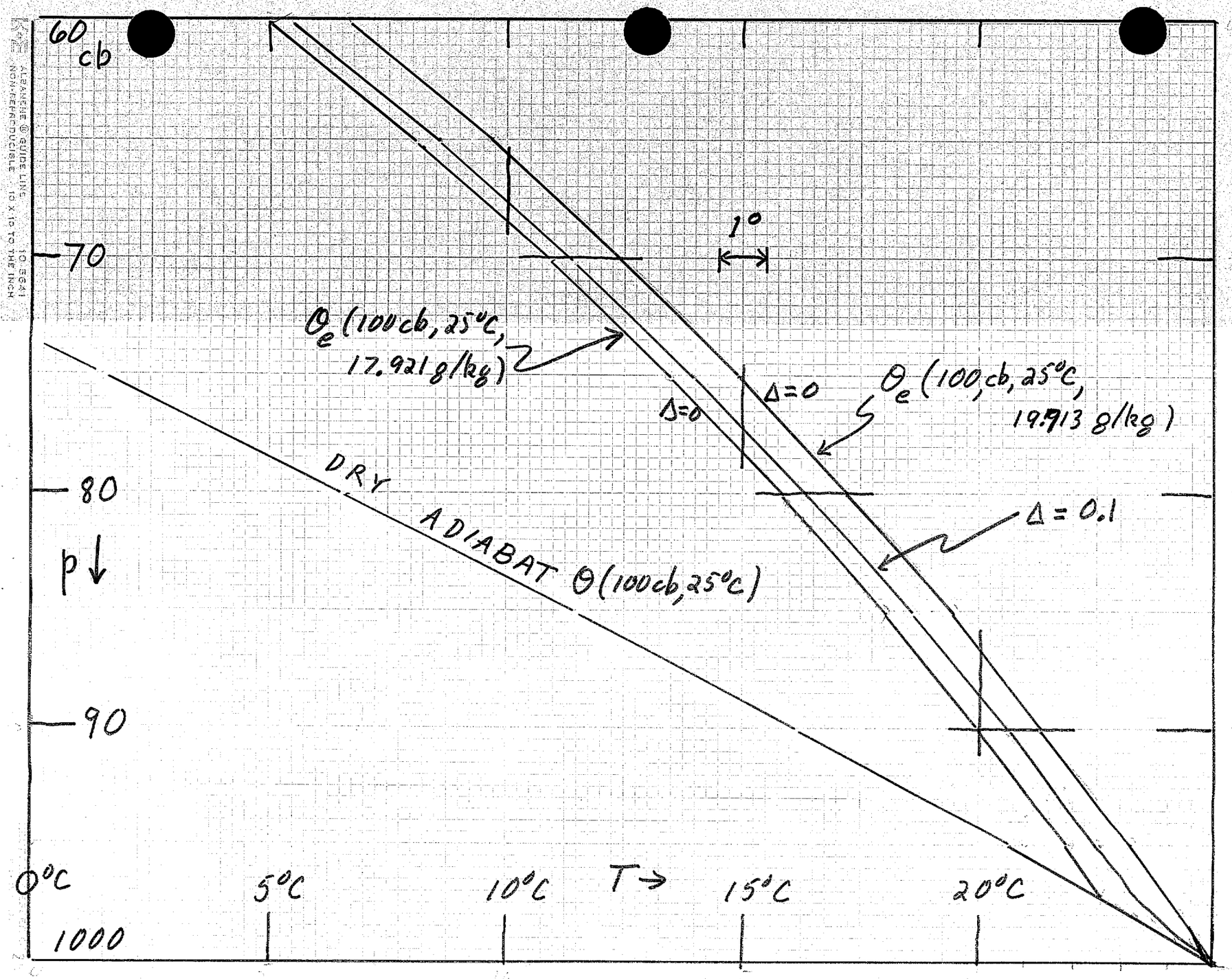

Fig. 1 Process curves for samples undergoing successive adiabatic pressure drops of 0.1 cb. The two outer curves are for $\Delta = 0$ computations starting with 90% and 100% relative humidity. The intermediate curve is for $\Delta = 0.1$ starting with $\bar{q}_0$ = 90% of q_s. A dry adiabat is shown for reference. The warmest $\Delta = 0$ curve is suggested as the appropriate "cloud" curve for Kuo convection parameterization.

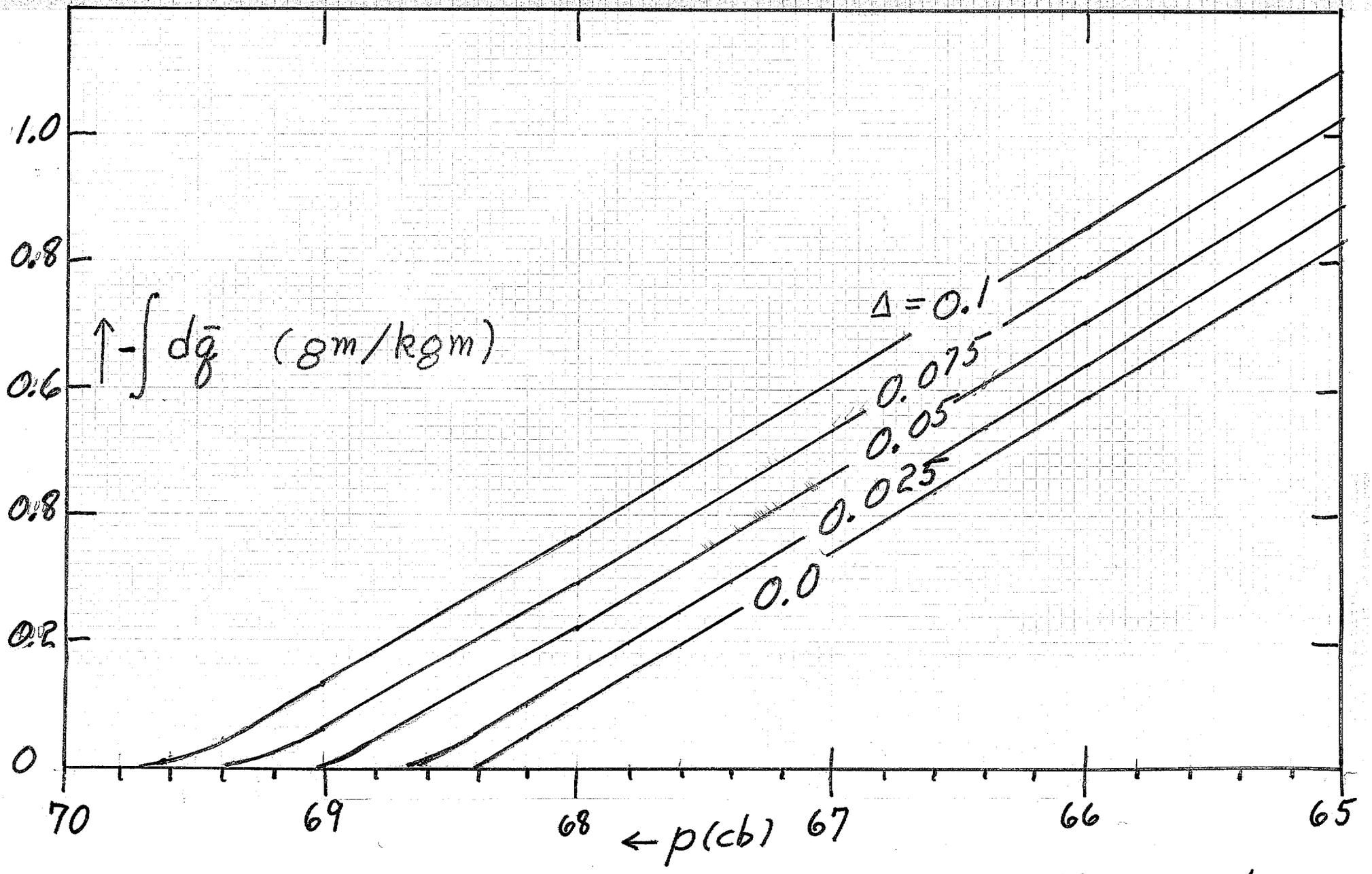

Fig. 2. Accumulated condensate for 5 values of Δ experienced by a sample starting at 70 cb with $T = 10°C$ and a relative humidity of 90%. ($\bar{q} = .009873$) when subjected to successive adiabatic pressure drops of 0.02 cb.

Table I. $\delta p = 1\,cb.$

		$\Delta = 0.0$	$\Delta = 0.025$		$\Delta = 0.05$		$\Delta = 0.075$		$\Delta = 0.1$	
P	T(°C)	$\bar{q}$	dT	$d\bar{q}$	dT	$d\bar{q}$	dT	$d\bar{q}$	dT	$d\bar{q}$
99	24.145264	0.017921	0.0	0.0	0.0	0.0	0.002930	-0.000001	0.032959	-0.000013
98	23.234180	0.017921	0.000977	-0.000000	0.045898	-0.000018	0.111328	-0.000045	0.197998	-0.000080
97	22.666504	0.017821	0.002197	-0.000001	0.065186	-0.000026	0.159668	-0.000065	0.271240	-0.000109
96	22.307861	0.017614	0.0	-0.000000	0.009277	-0.000004	0.081299	-0.000033	0.187256	-0.000076
95	21.942871	0.017405	0.0	-0.000000	0.003906	-0.000002	0.059326	-0.000024	0.155029	-0.000063
94	21.573486	0.017196	0.0	-0.000000	0.002930	-0.000001	0.052490	-0.000022	0.141602	-0.000058
93	21.199463	0.016985	0.0	-0.000000	0.002441	-0.000001	0.049072	-0.000020	0.134521	-0.000055
92	20.820557	0.016772	0.0	-0.000000	0.001953	-0.000001	0.046631	-0.000019	0.129639	-0.000053
91	20.436768	0.016558	0.0	-0.000000	0.001709	-0.000001	0.044434	-0.000019	0.125488	-0.000052
90	20.048096	0.016342	0.0	-0.000000	0.001221	-0.000000	0.042236	-0.000018	0.121338	-0.000050
89	19.654053	0.016125	0.0	0.0	0.000977	-0.000000	0.040283	-0.000017	0.117432	-0.000049
88	19.254883	0.015906	0.0	0.0	0.000732	-0.000000	0.038086	-0.000016	0.113525	-0.000048
87	18.850098	0.015686	0.0	0.0	0.000488	-0.000000	0.036133	-0.000015	0.109619	-0.000046
86	18.439697	0.015464	0.0	0.0	0.000244	-0.000000	0.033936	-0.000015	0.105713	-0.000045
85	18.023438	0.015240	0.0	0.0	0.000244	-0.000000	0.031982	-0.000014	0.102051	-0.000043
84	17.601318	0.015014	0.0	0.0	0.0	0.000000	0.030029	-0.000013	0.098145	-0.000042
83	17.173096	0.014787	0.0	0.0	0.0	0.000000	0.027832	-0.000012	0.094238	-0.000041
82	16.738281	0.014559	0.0	0.0	0.0	0.000000	0.026123	-0.000012	0.090576	-0.000039
81	16.297119	0.014328	0.0	0.0	0.0	0.000000	0.024170	-0.000011	0.086670	-0.000038
80	15.849121	0.014096	0.0	0.0	0.0	0.000000	0.022217	-0.000010	0.082764	-0.000036
79	15.394043	0.013862	0.0	0.0	0.0	0.000000	0.020508	-0.000010	0.079102	-0.000035
78	14.931885	0.013627	0.0	0.0	0.0	0.000000	0.018799	-0.000009	0.075439	-0.000034
77	14.462158	0.013389	0.0	0.0	0.0	0.000000	0.017090	-0.000008	0.071777	-0.000032
76	13.984619	0.013150	0.0	0.0	0.0	0.0	0.015625	-0.000008	0.068115	-0.000031
75	13.499268	0.012910	0.0	0.0	0.0	0.0	0.013916	-0.000007	0.064453	-0.000030
74	13.005615	0.012667	0.0	0.0	0.0	0.0	0.012207	-0.000007	0.060791	-0.000028
73	12.503174	0.012423	0.0	0.0	0.0	0.0	0.010742	-0.000006	0.057373	-0.000027
72	11.991943	0.012177	0.0	0.0	0.0	0.0	0.009521	-0.000005	0.053955	-0.000026
71	11.471436	0.011929	0.0	0.0	0.0	0.0	0.008301	-0.000005	0.050537	-0.000024
70	10.941406	0.011679	0.0	0.0	0.0	0.0	0.007080	-0.000004	0.047119	-0.000023
69	10.401611	0.011428	0.0	0.0	0.0	0.0	0.005615	-0.000004	0.043457	-0.000022
68	9.851318	0.011174	0.0	0.0	0.0	0.0	0.004395	-0.000003	0.040283	-0.000020
67	9.290283	0.010920	0.0	0.0	0.0	0.0	0.003418	-0.000003	0.036865	-0.000019
66	8.717773	0.010663	0.0	0.0	0.0	0.0	0.002686	-0.000003	0.033936	-0.000018
65	8.133789	0.010404	0.0	0.0	0.0	0.0	0.001709	-0.000002	0.030762	-0.000017
64	7.537598	0.010144	0.0	0.0	0.0	0.0	0.000977	-0.000002	0.027832	-0.000016
63	6.928711	0.009883	0.0	0.0	0.0	0.0	0.000488	-0.000002	0.024902	-0.000015
62	6.306641	0.009619	0.0	0.0	0.0	0.0	-0.000244	-0.000001	0.021973	-0.000014
61	5.670654	0.009354	0.0	0.0	0.0	0.0	-0.000732	-0.000001	0.019043	-0.000012
60	5.020020	0.009088	0.0	0.0	0.0	0.0	-0.000977	-0.000001	0.016602	-0.000011
59	4.354248	0.008820	0.0	0.0	0.0	0.0	-0.001221	-0.000001	0.014160	-0.000010
58	3.672607	0.008550	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.011719	-0.000009
57	2.974121	0.008280	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.009521	-0.000008
56	2.258301	0.008008	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.007080	-0.000008
55	1.523926	0.007734	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.005127	-0.000007
54	0.770020	0.007460	0.0	0.0	0.0	0.0	-0.001465	-0.000001	0.003174	-0.000006
53	-0.004150	0.007185	0.0	0.0	0.0	0.0	-0.001709	-0.000001	0.001221	-0.000005
52	-0.800049	0.006909	0.0	0.0	0.0	0.0	-0.001465	-0.000001	-0.000244	-0.000005
51	-1.618652	0.006632	0.0	0.0	0.0	0.0	-0.001465	-0.000001	-0.001709	-0.000004
50	-2.461182	0.006355	0.0	0.0	0.0	0.0	-0.001709	-0.000001	-0.002930	-0.000004

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