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Addition of Orography to the Semi-Implicit Version of the Shuman-Hovermale Model

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1. Introduction

The semi-implicit time integration scheme has been reported in the literature for several years and is used in a number of multilayer numerical weather prediction models around the globe. The implicit treatment permits a long time-step to be used in a forecast model because it time-averages terms in the equations which govern the fastest moving gravity waves.[1] All other terms are treated in the normal explicit sense. The computation time savings resulting from the long time-step make the semi-implicit technique particularly attractive for numerical models that are being used in an operational forecasting environment. Because a set of Helmholtz equations must be solved during each time step, the savings from the semi-implicit method is not so great as would be expected from the longer time step. However, computation time savings of four to one are reported for a six to one ratio of time step lengths in semi-implicit versus explicit runs.

A semi-implicit version of the Shuman-Hovermale 6-layer primitive equation model has been developed here at NMC by Gerrity, et al. (1973), and early experimental results without orography have been published by Campana (1974). Tests which included orography were initially unsuccessful, and it was with great difficulty that mountains were incorporated into the model. The purpose of this note is to document the solution to the mountain problem in the semi-implicit model. The first section will briefly describe the splitting of the equations

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[1]Pressure gradient term in the equations of motion and divergence term in the continuity equation.

into implicit and explicit parts. The next section will discuss the mountain problem and its solution. The final section will present the model equations which must be adjusted to fit the above solution. The actual model is not discussed in great detail, so the reader is referred to Gerrity (1973) for all the particulars.

2. Semi-Implicit Transformation

In order to discuss the mountain problem in the succeeding section, a brief description of the transformation of the equations of motion to semi-implicit time differencing is helpful. The equation of motion for the v component of the wind (equation (1)) is used for this discussion:

$$\frac{\partial}{\partial t}\frac{v}{m} + \frac{\partial \phi}{\partial y} + \alpha\frac{\partial p}{\partial y} = -\frac{\hat{f}u}{m} - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \frac{\dot{\sigma}}{m}\frac{\partial}{\partial \sigma} + \text{Friction} \qquad (1)$$

where

t = time

u = horizontal wind component in the x-direction

v = horizontal wind component in the y-direction

$\dot{\sigma}$ = vertical wind component in the σ-direction

p = pressure

α = specific volume

ϕ = geopotential

$\hat{f}$ = Coriolis and map factor terms = $f - v\frac{\partial m}{\partial x} + u\frac{\partial m}{\partial y}$

m = map factor

f = Coriolis force

Note that, unlike other models at NMC, this one uses temperature and pressure as the thermodynamic variables.

First, equation (1) is simplified by employing a linearization procedure.

Each variable, X, is assumed to be composed of a basic state, $\tilde{X}$, varying only with σ, and a deviation from this basic state, X'.

$$X = \tilde{X} + X' \tag{2}$$

Implicit calculations are done only on the resulting linear terms. Basic state values for the thermodynamic variables are obtained from the U.S. Standard Atmosphere (1962) using "representative" σ-layer pressures. The basic state wind field is one of no motion ($\tilde{u} = \tilde{v} = 0$).

Taking the $\alpha\frac{\partial p}{\partial y}$ term in equation (1) and defining

$$\alpha = \tilde{\alpha} + \alpha'$$

$$p = \tilde{p} + p' \quad \text{with } \frac{\partial \tilde{p}}{\partial y} = 0, \text{ since } \tilde{p} \text{ is a function of } \sigma \text{ only;}$$

one obtains

$$\alpha\frac{\partial p}{\partial y} = \tilde{\alpha}\frac{\partial p'}{\partial y} + \alpha'\frac{\partial p'}{\partial y} \tag{3}$$

In the semi-implicit treatment of equation (1), terms on the left side are time averaged (implicit calculation). Rewriting equation (1) using the linearization process for all terms except $\frac{\partial \phi}{\partial y}$, one obtains:

$$\frac{\partial}{\partial t}\frac{v'}{m} + \frac{\partial \phi}{\partial y} + \tilde{\alpha}\frac{\partial p'}{\partial y} = -\alpha'\frac{\partial p'}{\partial y} - \hat{f}\frac{u'}{m} - u'\frac{\partial v'}{\partial x} - v'\frac{\partial v'}{\partial y} - \frac{\dot{\sigma}'}{m}\frac{\partial v'}{\partial \sigma} + \text{Friction} \tag{4}$$

Note that the $\alpha'\frac{\partial p'}{\partial y}$ term is nonlinear and is calculated on the explicit (non-time averaged) side.

Letting a superscript, τ, denote quantities evaluated explicitly at time τ, the following definitions of the time average, $\overline{X}^{2t}$, and the time derivative, $\frac{\partial X}{\partial t}$, are useful when transforming equation (4) to its semi-implicit counterpart,

$$\overline{X}^{2t} = \tfrac{1}{2}(X^{\tau+1} + X^{\tau-1})$$

$$\frac{\partial X}{\partial t} = \frac{X^{\tau+1} - X^{\tau-1}}{2\Delta t} = \frac{\overline{X}^{2t} - X^{\tau-1}}{\Delta t}$$

Implicit treatment of the left side of equation (4) and dropping the primes from all variables leaves the following:

$$\frac{\overline{v}^{2t}}{m} + \Delta t\left(\frac{\partial\overline{\phi}^{2t}}{\partial y} + \tilde{\alpha}\frac{\partial\overline{p}^{2t}}{\partial\hat{y}}\right) = \frac{v^{\tau-1}}{m} + \Delta t(\ldots\ldots)^{\tau} \qquad (5)$$

where $(\ldots\ldots)^{\tau}$ represents all terms on the right side of equation (4).

In the actual model equations, σ-layer pressure thicknesses, $\partial p/\partial\sigma$, are used in the pressure gradient term, rather than pressure itself. Further, in order to close the system of equations, $\overline{\phi}^{2t}$ is transformed into implicit terms involving $\overline{\partial p}^{2t}/\partial\sigma$, $\overline{\dot{\sigma}}^{2t}$ and into other terms calculated explicitly, R. Replacing $\overline{\phi}^{2t}$ by these terms in equation (5), and using the real model variables, one obtains:

$$\frac{\overline{v_k}^{2t}}{m} + \frac{\partial}{\partial y}\sum_{j=1}^{3} g_{k,j}\left(\overline{\frac{\partial p}{\partial\sigma}}\right)_j^{2t} + \frac{\partial}{\partial y}\sum_{j=1}^{4}\hat{h}_{k,j}\,\overline{\dot{\sigma}}_j^{2t} = \frac{v_k^{\tau-1}}{m} - \Delta t\frac{\partial R_k^{\tau}}{\partial y} + \Delta t(\ldots.)_k^{\tau} \qquad (6)$$

k = vertical index

where $\frac{\partial R_k^{\tau}}{\partial y}$ and the matrices, $g_{k,j}$, $\hat{h}_{k,j}$ all result from the transformation of $\overline{\phi}^{2t}$ (see Section 4 in Gerrity, 1973). By solving a set of Helmholtz equations, one obtains the three $\overline{\partial p}^{2t}/\partial\sigma$ and the four $\overline{\dot{\sigma}}^{2t}$ which are needed to compute $\overline{v}^{2t}$ from equation (6).

The preceding general description of the semi-implicit transformation now allows one to proceed to a discussion of the orographic problem.

3. Orography

Semi-implicit model experiments without orography were quite successful using a time step of 1 hour. When mountains were introduced, however, erroneous orographic scale features developed over large mountain masses and amplified with time. An example of this problem over the Rockies and Himalayas

is shown in figure 1. Tests with lower mountain elevations only lessened the real difficulty. When the model was run in an entirely explicit mode (and thus a shorter time step) the problem disappeared (figure 2). Further tests with the semi-implicit version, using a time step as short as the explicit mode above, also yielded trouble-free forecasts. There appeared to be severe time truncation errors near orography when using a long time step.

After a great deal of reflection and experimentation, the problem appeared to be related to the implicit/explicit splitting of the pressure gradient term[2] in the tropospheric sigma domain. Recalling equation (1), the pressure gradient near mountains is made up of two relatively large terms having opposite signs $\left(\frac{\partial\phi}{\partial y}, \alpha\frac{\partial p}{\partial y}\right)$. Performing the semi-implicit transformation on this equation, these terms are further broken into implicit and explicit parts. Close examination shows that they also can be large terms of opposite sign in the vicinity of mountains. Since the basic state pressure, $\tilde{p}$, is not a function of (x,y), gradients of pressure near orography remain in the deviation part, p'. Thus a good portion of the large $\alpha\frac{\partial p}{\partial y}$ term near mountains remains on the implicit side of the equation in $\tilde{\alpha}\frac{\partial p'}{\partial y}$. However, the process of transforming $\overline{\phi}^{2t}$ leaves the gradient of ground elevation on the explicit side of equation (6) imbedded in the $\frac{\partial R_k}{\partial y}$ term. Examination of the two parts of the pressure gradient term at a grid point near steep mountains shows they both are larger than any other term in equation (6).

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[2]in the equations of motion.

Table 1 displays the size of these parts and their sum in the lowest tropospheric layer of the model during a semi-implicit forecast (1 hour time step). The sum amplifies with time and the implicit part seems to cause most of the increase. Only 11 forecast hours are needed to produce negative model pressures over the mountains.

Since the gradient of ground height and the gradient of model surface pressure are of opposite sign, the magnitude of both parts of the pressure gradient term can be reduced by calculating both of them on the same side of the equation, either explicitly or implicitly, rather than separately. In order to disrupt the model formulated by Gerrity (1973) as little as possible, a redefinition of the pressure deviation, p', is made in the troposphere,

$$p' = p'' + \tilde{p}' \tag{7}$$

where $\tilde{p}'$ is a surface pressure at the top of the model mountains obtained from the U.S. Standard Atmosphere. All parts of equation (7) are functions of (x,y) and $\tilde{p}'$ is the time invariant. Now redefine the implicit term,

$$\tilde{\alpha}\frac{\partial p'}{\partial y} = \tilde{\alpha}\frac{\partial p''}{\partial y} + \tilde{\alpha}\frac{\partial \tilde{p}'}{\partial y} \tag{8}$$

Moving the time invariant quantity, $\tilde{\alpha}\frac{\partial \tilde{p}'}{\partial y}$ to the explicit side of equation (4) one obtains

$$\frac{\partial}{\partial t}\frac{v'}{m} + \frac{\partial \phi}{\partial y} + \tilde{\alpha}\frac{\partial p''}{\partial y} = -\alpha'\frac{\partial p'}{\partial y} - \frac{\hat{f}u'}{m} - u'\frac{\partial v'}{\partial x} - v'\frac{\partial v'}{\partial y} - \frac{\dot{\sigma}'}{m}\frac{\partial v'}{\partial \sigma} + \text{Friction} - \tilde{\alpha}\frac{\partial \tilde{p}'}{\partial y} \tag{9}$$

In essence the tropospheric basic state pressure is adjusted to account for orography. Recalling that σ-layer pressure thickness, $\partial p/\partial \sigma$, is used rather than pressure, p, in the actual model, equation (6) in the troposphere

(k = 4,5,6,7) becomes:

$$\frac{\overline{v_k}^{2t}}{m} + \frac{\partial}{\partial y} \sum_{j=1}^{3} g_{k,j} \overline{\left[\frac{\partial p}{\partial \sigma}\right]}_j^{2t} + \frac{\partial}{\partial y} \sum_{j=1}^{4} \hat{h}_{k,j} \overline{\dot{\sigma}}_j^{2t} = \frac{v_k^{\tau-1}}{m} - \Delta t \frac{\partial R_k^{\tau}}{\partial y} + \Delta t(\ldots.)_k^{\tau}$$

$$- \Delta t\, \tilde{\alpha}_k \frac{\partial \tilde{p}'}{\partial y} \qquad (10)$$

Of course in a like manner there is a $\frac{\partial \tilde{p}'}{\partial x}$ term in the u-equation of motion.

This redefinition of the deviation part of the pressure variable and its proper splitting into implicit and explicit parts removed the amplifying mountain features. Successful semi-implicit forecasts using an hour time step have been made beyond 48 hours. Examination of the two parts of the pressure gradient term at one grid point in Table 2 shows them to be an order of magnitude smaller with the above modification than with the old formulation (Table 1). The implicit part, which seemed responsible for the amplification, is now under control.

4. Changes to Model Equations

This section documents changes in the actual semi-implicit model equations that are necessary to remove the mountain problem. Gerrity (1973) denotes sigma domain pressure thicknesses as π, so equation (7) becomes

$$\pi_k' = \pi_k'' + \tilde{p}' \quad \text{for } k = 3 \qquad (11)$$

where k = 3 is the tropospheric sigma domain. Notice that $(\tilde{\ })$ refers to basic state variables, that the primes on the deviation parts are dropped ($\pi_3' \equiv \pi_3$), and that the $\tilde{p}'$ notation for the standard atmosphere surface pressure at the mountain tops is retained. Changed model equations are presented below, where equation numbers noted are those of Gerrity (1973):

1. Equation (83) becomes:

$$\vec{V}_k^\tau = \frac{\vec{v}_k^{\tau-1}}{m} - \Delta t[(\alpha_k^\tau - \tilde{\alpha}_k)\hat{\nabla}(\alpha_k\pi_3^\tau + \pi_2^\tau + \pi_1^\tau) + \frac{\hat{f}_k^\tau}{m}\vec{k} \times \vec{v}_k^\tau$$

$$+ \vec{v}_k^\tau \cdot \hat{\nabla}\vec{v}_k^\tau + B_k^\tau + \tilde{\alpha}_k\sigma_k\hat{\nabla}\tilde{p}']$$

2. Equation (116) becomes:

$$\vec{V}_7^\tau = \frac{\vec{v}_7^{\tau-1}}{m} - \Delta t[(\alpha_7^\tau - \tilde{\alpha}_7)\hat{\nabla}(\pi_3^\tau + \pi_2^\tau + \pi_1^\tau) + \frac{\hat{f}_7^\tau}{m}\hat{k} \times \vec{v}_k^\tau$$

$$+ \vec{v}_7^\tau \cdot \hat{\nabla}\vec{v}_7^\tau + B_7^\tau - \vec{F} + \tilde{\alpha}_7\hat{\nabla}\tilde{p}']$$

Changes also must be made to other equations which contain π_3:

3. Equation (107) becomes:

$$P_T^\tau = \pi_3^{\tau-1} + \Delta t\ \nabla\cdot[(\tilde{\pi}_3 - p_c)\vec{v}_T^\tau] - \Delta t\ \nabla\cdot[(\pi_3^\tau - p_c)\vec{v}_T^\tau] - \tilde{p}'$$

4. Equation (112) becomes:

$$G_k^\tau = \tilde{\alpha}_k p_c - (\alpha_k^\tau - \tilde{\alpha}_k)(\pi_3^\tau - \tilde{\pi}_3) - \tilde{\alpha}_k\tilde{p}'$$

5. Equation (114) becomes:

$$I_k^\tau = \tilde{\alpha}_k p_c - (\alpha_k^\tau - \tilde{\alpha}_k)[\pi_3^\tau + \frac{1}{\sigma_k}(\pi_2^\tau + \pi_1^\tau) - \tilde{\pi}_3 - \frac{1}{\sigma_k}(\tilde{\pi}_2 + \tilde{\pi}_1)] - \tilde{\alpha}_k\tilde{p}'$$

6. Equation (125) becomes:

$$I_7^\tau = \tfrac{1}{2}\tilde{\alpha}_7 p_c - [(\pi_1^\tau + \pi_2^\tau + \pi_3^\tau - \tilde{\pi}_1 - \tilde{\pi}_2 - \tilde{\pi}_3)(\alpha_7^\tau - \tilde{\alpha}_7)] - \tilde{\alpha}_7\tilde{p}'$$

Changes also have to be made to the Helmholtz equations, since the tropospheric pressure thickness, π_3, on the implicit side of the equations has been changed to π_3'' through equation (11).

7. Equation (236) becomes:

$$P^T = \{\bar{\pi}_1^{2t}, \bar{\pi}_2^{2t}, \overline{(\pi_3 - \tilde{p}')}^{2t}, \bar{w}_1^{2t}, \bar{w}_2^{2t}, \bar{w}_3^{2t}, \bar{w}_4^{2t}\}$$

REFERENCES

Campana, K. A., 1974: "Status Report on a Semi-Implicit Version of the Shuman Hovermale Model." NOAA Technical Memorandum NWS-NMC-54, U.S. Dept. of Commerce, Washington, DC, 22 pp.

Gerrity, J. P., R. D. McPherson, and S. H. Scolnik, 1973: "A Semi-Implicit Version of the Shuman Hovermale Model." NOAA Technical Memorandum NWS NMC-53, U.S. Department of Commerce, Washington, DC, 44 pp.

Table 1. Implicit and explicit parts of pressure gradient term in equation (6) at one grid point (k=6) - units in m/sec - expressed as effect on $\overline{v}_6^{2t}/m$.

FCST HR	PRESSURE GRADIENT (Implicit)	PRESSURE GRADIENT (Explicit)	TOTAL PRESSURE GRADIENT
1	+14.5	-13.2	+1.3
2	+14.7	-13.2	+1.5
3	+14.8	-13.1	+1.7
4	+14.9	-13.2	+1.7
5	+15.0	-13.1	+1.9
6	+15.0	-13.1	+1.9
7	+15.2	-13.0	+2.2
8	+15.4	-13.0	+2.4
9	+15.8	-12.9	+2.9
10	+16.3	-12.7	+3.6
11	+16.9	-12.6	+4.3

Table 2. Implicit and explicit parts of pressure gradient term in equation (10) at one grid point (k=6) - units in m/sec - expressed as effect on $\overline{v}_6^{2t}/m$.

FCST HR	PRESSURE GRADIENT (Implicit)	PRESSURE GRADIENT (Explicit)	TOTAL PRESSURE GRADIENT
1	- .7	+1.1	+ .4
2	- .8	+1.1	+ .3
3	- .8	+1.1	+ .3
4	- .7	+1.1	+ .4
5	- .6	+1.2	+ .6
6	- .6	+1.2	+ .6
7	- .6	+1.2	+ .6
8	- .7	+1.1	+ .4
9	- .7	+1.1	+ .4
10	- .7	+1.1	+ .4
11	- .7	+1.1	+ .4

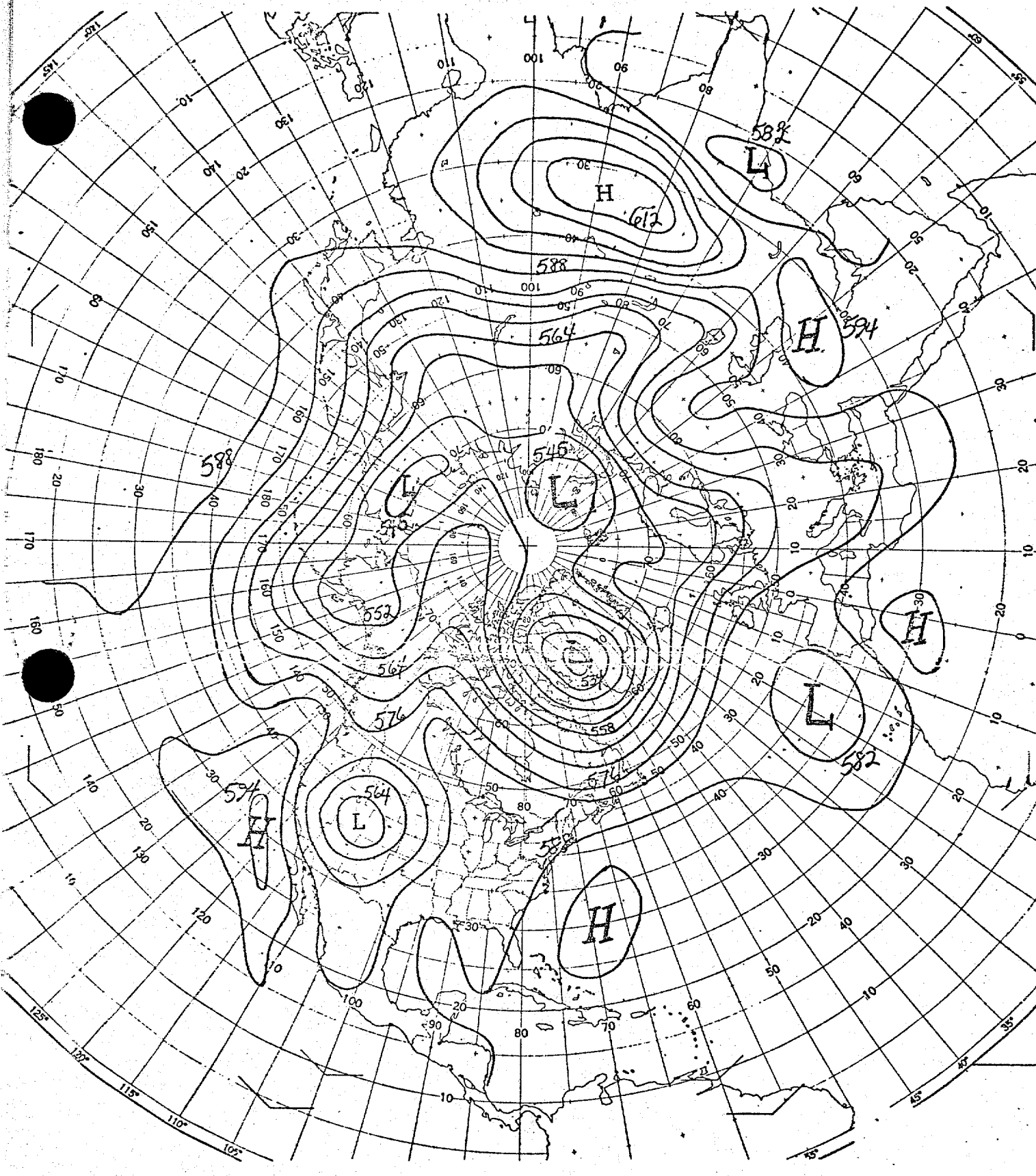

Figure 1. Semi-Implicit Forecast. Time step = 3600 sec. 500 mb heights.

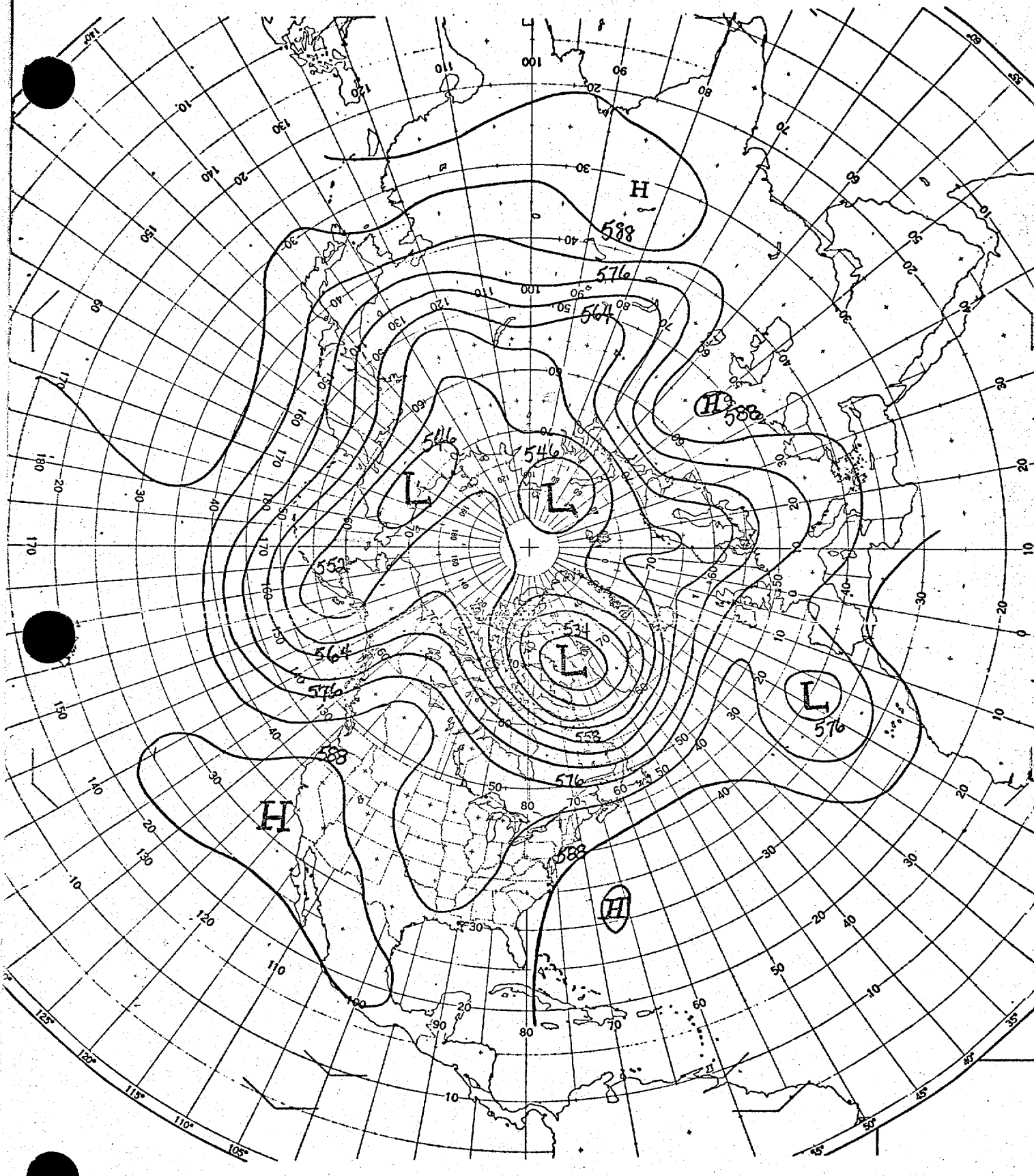

Figure 2. Explicit forecast. Time step = 600 sec 500 mb heights