U. S. DEPARTMENT OF COMMERCE
ENVIRONMENTAL SCIENCE SERVICES ADMINISTRATION
WEATHER BUREAU

NATIONAL METEOROLOGICAL CENTER

OFFICE NOTE NO. 32

ON A SPECIAL FORM FOR VISCOUS TERMS

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December 10, 1969

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Shuman and Stackpole (1969) proposed special forms for the damping terms, viz.,

$$\frac{\partial u}{\partial t} + \ldots - \mu \frac{\partial D}{\partial x} + \nu \frac{\partial \zeta}{\partial y} = 0$$

$$\frac{\partial v}{\partial t} + \ldots - \mu \frac{\partial D}{\partial y} - \nu \frac{\partial \zeta}{\partial x} = 0$$

where D and ζ are divergence and vorticity. The divergence and vorticity equations derived from these are

$$\frac{\partial D}{\partial t} + \ldots - \mu \nabla^2 D = 0$$

$$\frac{\partial \zeta}{\partial t} + \ldots - \nu \nabla^2 \zeta = 0$$

With these special forms of viscosity, therefore, vorticity ζ and divergence D may be selectively damped without directly affecting one another.

Possible uses of these are

1. With $\nu = 0$, the gravitational oscillation could be damped during a prediction without directly affecting the vorticity, with which the Rossby modes are associated.

2. Direct damping of divergence (again with $\nu = 0$) could be used with marching methods of initialization, instead of high frequency dampers such as the Euler-backward scheme.

3. As a special case of 2., a solution of the balance equation could be obtained by marching with the barotropic primitive equations, somehow maintaining or periodically restoring the height field.

After Shuman and Stackpole (1969), when Shuman and McGovern attempted to use the special forms in a calculation, they realized that they could not readily be formulated to heavily damp all of the highest spatial frequencies. They used a coupled grid, however, and their difficulty vanishes in the decoupled grid which has been discussed lately in Development Division, and was an outgrowth of Robert, Shuman, and Gerrity (1970).

Specific forms for the suggested terms are

$$u_{2t} + \ldots - (\mu D_{2x} - \nu \zeta_{2y})_{n-1} = 0$$

$$v_{2t} + \ldots - (\mu D_{2x} + \nu \zeta_{2y})_{n-1} = 0$$

where

$$D = u_{2x} + v_{2y}$$

$$\zeta = v_{2x} - u_{2y}$$

and n is the time index.

Figure 1 shows the arrangement of the grid. The "even" points are shown by intersections of the diagonal lines, and are at "even" time levels, defined by n being even. The "odd" points are shown by "+'s", and are at "odd time levels, defined by n being odd.

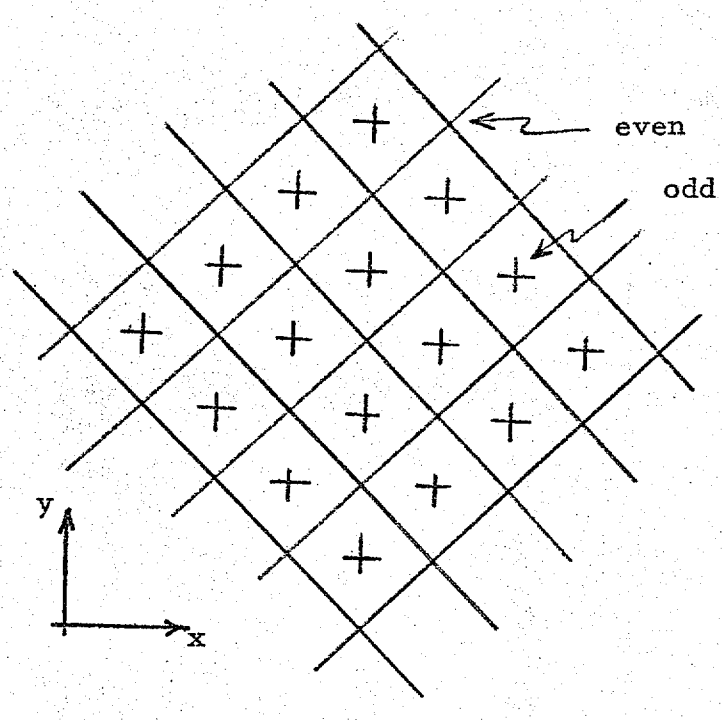

Figure 1.

References

Robert, A. J., F. G. Shuman, and J. P. Gerrity, 1970: On the partial differential equations of mathematical physics. Mon. Wea. Rev., V. 98, No. 1, to be published.

Shuman, F. G., and J. D. Stackpole, 1969: The currently operational NMC model, and results of a recent numerical experiment. Proc. WMO/IUGG Symp. Numerical Weather Prediction, Tokyo, Japan Meteorological Agency, II-85-98.